

Risk-Adjusted Payback Period Valuation: A Practical Alternative to Traditional DCF Models

Sangyul Baek

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Abstract

This paper introduces the Risk-Adjusted Payback Period (RAPP) model, an equity valuation methodology that bridges traditional discounted cash flow (DCF) analysis and the payback-based heuristics that practitioners have long gravitated toward. RAPP values a stock by summing undiscounted cumulative earnings per share (or free cash flow per share for capital-intensive sectors) over a calibrated time horizon, and proves that this undiscounted sum exactly equals the DCF fair value when the horizon is set by the Discount Rate–Payback Period Mapping (DRPM):

$$N^* = \ln(r / (r - g)) / \ln(1 + g)$$

where r is the CAPM-derived required return and g is the long-run growth rate. The key theoretical insight is that at N^* , DCF's two core adjustments—discounting and terminal value—cancel exactly, making discount factors unnecessary rather than merely simplified.

The paper makes three principal contributions. First, it provides the formal equivalence proof connecting DRPM to CAPM, replacing discretionary horizon assignment with a closed-form mapping from observable market parameters. Unlike DCF, which requires an external growth ceiling to prevent terminal value blowup, RAPP's logarithmic structure handles high growth rates gracefully—the only hard constraint is $g < r$ —with practical guidance for terminal growth estimation through the sector excess growth rate (α) framework. Second, it demonstrates that RAPP's logarithmic response to the r - g singularity is structurally more robust than DCF's hyperbolic blowup: the sensitivity ratio between DCF and RAPP grows monotonically as the spread narrows, reaching $2.3\times$ at a moderate r - g of 7% and exceeding $21\times$ at r - $g = 0.5\%$. Third, it introduces two derivative metrics: Payback Duration, a risk-weighted recovery measure connecting to the equity duration literature (Dechow, Sloan & Soliman, 2004; Lettau & Wachter, 2007), and Implied Payback Period, a market sentiment diagnostic that translates a stock's current price into the beta the market is implicitly assigning. Three complementary inversions—Implied α , Implied N^* , and Break-even α —extend this diagnostic into a comprehensive framework for evaluating high-growth firms where terminal growth assumptions materially affect valuation.

Behavioral grounding comes from a two-round thought experiment with roughly 50 finance professionals, where respondents in this convenience sample translate risk directly into a shorter implied payback horizon rather than computing a higher discount rate. Case studies on Carvana, Tesla, and Palantir demonstrate 1–3% convergence with DCF for properly calibrated cases.

Keywords: Equity Valuation, Payback Period, DCF Alternative, Discount Rate Mapping, Equity Duration, Implied Payback Period

1. Introduction

1.1 Motivation

1.1.1 A Thought Experiment Imagine a goose that lays golden eggs. Each egg is currently worth \$100,000, and the goose produces exactly one egg per year. If you were buying this goose, what is the most you would pay? If you already owned it, what is the least you would accept to sell?

Over the past three years, roughly 50 finance professionals—colleagues, friends, and investors—have been posed these two questions. No price range was suggested; respondents simply named a dollar amount. After collecting initial answers, a second round introduced risk: the price of gold could fluctuate (or, since gold prices have risen over the past decade, they could keep rising), the goose could get sick, its lifespan is uncertain, and the eggs might vary in size.

Every respondent in this sample lowered their price once uncertainty was introduced. The predominant response to risk was to demand fewer years of earnings exposure—to shorten the implied payback horizon. After converting dollar responses to implied payback years (dividing by \$100,000), the results from the risk-adjusted round showed buyer willingness-to-pay clustering between \$800,000 and \$1,100,000 (8–11 years), while seller reservation prices clustered between \$1,000,000 and \$1,300,000 (10–13 years). The overlap zone corresponds to roughly 9 to 11 years, with the central tendency near 10.

This bid-ask spread behaves exactly as the endowment effect predicts (Thaler, 1980; Kahneman et al., 1991). While this constitutes a limited rather than controlled academic sample, the consistency of the clustering provides meaningful behavioral evidence; formal large-scale validation is identified as a priority for future research (Section 6.6).

The mechanism, not the specific numbers, is the central observation. Nobody was told to think in payback periods or given a discount rate, yet respondents translated risk into a lower price—mathematically equivalent to demanding a shorter payback horizon. That the resulting range (8–13 years) aligns with CAPM-implied payback horizons for typical equities (a 10% required return implies $1/r = 10$ years) suggests this pattern may reflect a cognitive heuristic rather than coincidence, though formal validation is required to establish this (Section 6.6).

The mechanism demonstrated here—translating risk into shorter payback horizons rather than computing discount rates—is consistent with behavioral finance principles (Kahneman, 2011; Simon, 1955) and the practitioner preference for payback methods documented in capital budgeting surveys (Graham & Harvey, 2001; Pike, 1996). That the resulting range aligns with the CAPM-implied horizons for equities with required returns of 7.7–12.5% suggests payback-based reasoning is a fundamental cognitive heuristic for valuing cash-flow-generating assets under uncertainty.

1.1.2 The Theory-Practice Tension There is a persistent tension in equity valuation. On one side, the DCF model is praised by textbooks for its theoretical purity, refined by Koller et al. (2020) and Damodaran (2012). On the other side, practitioners keep reaching for simpler tools. Survey after survey documents this: Graham and Harvey (2001) found 56.7% of CFOs used payback period for capital budgeting; Pike (1996) documented its enduring popularity in the UK over nearly two decades; Brounen et al. (2004) documented the same across Europe; Ryan and Ryan (2002) found

over half of Fortune 1000 firms regularly using payback. These are not unsophisticated people. Something about payback resonates.

The trouble with DCF is well-documented, even by its proponents. Terminal value often makes up 60–80% of total firm value (Copeland et al., 2000). Nudge the perpetual growth rate by half a percent and the output shifts dramatically—a phenomenon Damodaran (2006) called “precision without accuracy.” The most fundamental vulnerability is the singularity problem: the Gordon Growth terminal value $TV = FCF/(r - g)$ contains the denominator $r - g$, which approaches zero as the growth rate approaches the discount rate. At $r - g = 1\%$, the terminal value is $100 \times FCF$; at $r - g = 0.5\%$, it is $200 \times FCF$. The sensitivity is hyperbolic: $\partial TV/\partial g \propto 1/(r-g)^2$, meaning small errors in the growth assumption produce enormous valuation swings precisely where the assumption is hardest to estimate. Section 3.4.6 demonstrates that RAPP resolves this structurally.

But the deeper issue, illuminated by the goose experiment, is cognitive. DCF requires specifying a discount rate (itself combining risk-free rates, betas, and market risk premiums), then mechanically discounting every future cash flow, then adding a terminal value assuming steady-state economics extending to infinity. Each step is theoretically defensible, but the edifice feels disconnected from how investors actually assess value. The goose experiment suggests people intuitively ask “over what time horizon am I comfortable waiting for this to pay me back?”—a fundamentally different and more answerable question than “what discount rate should I apply to these cash flows?”

1.1.3 From Thought Experiment to Formal Framework This paper formalizes the payback concept for equity valuation. The Risk-Adjusted Payback Period (RAPP) model asks “how many years of future earnings justify the current stock price?” Risk enters through variable time horizons: riskier firms get shorter acceptable periods, stable ones get longer. A closed-form formula (the DRPM, derived in Section 3.4) computes the appropriate horizon directly from the same CAPM inputs used in DCF. The intuitive appeal matches how people naturally think about value; the formal surprise is that this approach produces valuations within 1–3% of traditional DCF.

1.2 Research Contribution

This paper contributes to the valuation literature in eight ways.

First, behavioral grounding for the payback horizon approach through the golden-egg goose thought experiment. In a two-round design, approximately 50 finance professionals first price a goose producing \$100,000 per year with no risk information, then reprice after uncertainty is introduced. Every respondent in this sample lowers their price, producing risk-adjusted valuations implying 8–13 years of current earnings—aligning with CAPM-implied payback horizons. The buyer-seller asymmetry is consistent with the endowment effect (Thaler, 1980). The common mechanism observed across this sample—translating risk into a shorter payback horizon rather than a higher discount rate—is consistent with payback-based thinking being a cognitive heuristic for assessing value under uncertainty, though formal large-scale validation remains a priority for future research (Section 6.6).

Second, mathematical proof that undiscounted cumulative earnings over finite horizons equal DCF valuations when the horizon is correctly calibrated. The core insight is that the effects of discounting and terminal value offset each other when the horizon is set by the DRPM formula—a condition that holds for a wide range of real companies (Appendix B).

Third, derivation of the DRPM formula, a closed-form mapping that computes the optimal payback

horizon directly from the required return and growth rate. This connects to the equity duration literature (Dechow, Sloan & Soliman, 2004; Lettau & Wachter, 2007; Weber, 2018) and introduces Payback Duration and Implied Payback Period as derivative metrics.

Fourth, distinction from Sam's (2025a-f) Potential Payback Period (PPP) methodology, which uses the identical formula for relative ranking rather than absolute valuation. The DRPM formula and PPP formula are mathematically identical, as both solve the same underlying equation; the key difference is the convergence proof (Appendix B) establishing that discounting is unnecessary at N^* .

Fifth, application to three diverse case studies demonstrating versatility across business models from high-growth turnarounds (Carvana) to mature growth with optionality (Tesla) to high-operating-leverage platforms (Palantir).

Sixth, demonstration that RAPP provides a structural resolution to the terminal value singularity problem, with sensitivity advantages of up to $21 \times$ over DCF at narrow spreads. Existing remedies—the Value Driver Formula (Koller et al., 2020), fade models (Holland, 2018), the H-Model (Fuller & Hsia, 1984)—all operate within the $1/(r-g)$ structure. RAPP escapes it entirely: the DRPM formula maps the same inputs to a logarithmic output space where the singularity manifests as a long but finite and interpretable time horizon rather than an unbounded valuation multiple.

Seventh, a two-tier cash flow framework (EPS as default, FCF for capital-intensive sectors) covering the full equity universe, with a concrete switching rule grounded in the accrual accounting literature (Dechow, 1994; Sloan, 1996).

Eighth, a sector excess growth rate (α) framework that provides systematic guidance for terminal growth estimation, with practical tools for conglomerate treatment through weighted-average α and scenario-based valuation for nascent industries. Three complementary inversions—Implied α , Implied N^* (α fixed), and Break-even α —transform RAPP from a single-point valuation into a comprehensive market belief diagnostic for high-growth firms.

1.3 Paper Structure

Section 2 reviews the relevant literature, including equity duration research. Section 3 develops the theoretical framework. Section 4 describes the methodology. Section 5 presents case studies. Section 6 discusses implications and limitations. Section 7 concludes.

2. Literature Review

2.1 Traditional Valuation Methods

The standard DCF framework values equity as $V_0 = \sum_{t=1}^T FCF_t / (1+WACC)^t + TV / (1+WACC)^T$. Terminal value is typically calculated using either the perpetuity growth method or exit multiples. Copeland et al. (2000) documented that terminal value represents 70–80% of total DCF value with 5-year forecasts—a remarkable concentration of value in the least observable component of the model. The perpetuity growth method is vulnerable to the terminal value singularity: as g approaches r , terminal value grows without bound. The Value Driver Formula (Koller et al., 2020), fade models (Holland, 2018), and multi-stage DDMs (Fuller & Hsia, 1984; Damodaran, 2012) offer partial mitigation but do not alter the fundamental $1/(r-g)$ structure.

DCF's sensitivity to input assumptions is well-documented. For a typical growth company ($r = 10\%$, $g = 3\%$), a 1-percentage-point change in the perpetual growth rate shifts terminal value by

approximately 17%. For narrower spreads, the sensitivity intensifies nonlinearly: at $r - g = 2\%$, the same 1-percentage-point change in g doubles the terminal value (from $50\times$ to $100\times$ FCF). This fragility is not an implementation problem but a structural property of the $1/(r-g)$ form.

The structural limitation of relative valuation also deserves emphasis. Comparable company analysis answers “is this stock cheap or expensive relative to peers?” rather than “what is this stock worth?” Comps inherit whatever mispricing characterizes the peer group. In episodes of sector-wide overvaluation—technology stocks in 2021, homebuilders in 2006—relative valuation fails to identify that the sector itself is overpriced. RAPP anchors fair value to fundamental EPS projections rather than market sentiment.

2.2 The Theory-Practice Gap

The survey evidence on practitioner behavior is remarkably consistent across countries and decades. Graham and Harvey (2001) found payback at 56.7%; Brounen et al. (2004) documented the same across Europe; Block (2007) confirmed persistent use in the US. The explanations cluster around simplicity and communication value (Lefley, 1996), liquidity concerns (Gitman & Forrester, 1977), implicit risk aversion (Boardman et al., 1982), and real options reasoning (Trigeorgis, 1996).

The academic critique—ignoring post-payback cash flows, neglecting time value of money—applies to payback in capital budgeting, where the investment is fixed and the question is whether it will be recovered. The question this paper asks is whether payback logic can work for equity valuation, where the mathematics differ in a critical way: the investment (stock price) is the unknown, and the payback period is calibrated to ensure theoretical consistency with DCF. The traditional critique assumes the payback period is exogenous; the DRPM formula makes it endogenous to the required return and growth rate, resolving both objections simultaneously.

2.3 Equity Duration and Cash Flow Timing

A substantial literature has developed around the concept of equity duration, which bears directly on RAPP’s theoretical framework. Dechow, Sloan, and Soliman (2004) introduced implied equity duration as a measure of equity risk, constructing a price-implied duration metric analogous to Macaulay bond duration. They demonstrated that stock price volatility and beta are both positively correlated with equity duration, and that their duration measure subsumes the Fama and French (1993) book-to-market factor in stock returns. This finding is directly relevant to RAPP’s Payback Duration metric (Section 3.5), which formalizes a closely related concept within the payback valuation framework.

Lettau and Wachter (2007) provided a theoretical model in which the value premium is explained by the short cash-flow duration of value firms. Their framework predicts that assets with shorter-duration cash flows (value stocks) should earn higher expected returns than assets with longer-duration cash flows (growth stocks), because short-duration assets are more exposed to shocks in expected returns. This duration-based explanation of the value premium provides theoretical support for RAPP’s central mechanism: converting risk into shorter payback horizons is consistent with the equilibrium prediction that higher-risk (higher required return) assets are associated with shorter cash-flow duration.

Weber (2018) extended this line of research by demonstrating that cash flow duration predicts stock returns in the cross-section, with short-duration stocks earning a significant premium over

long-duration stocks—approximately 1.1% per month between extreme deciles. Gonçalves (2021) confirmed and extended this finding, demonstrating that the short-duration premium is long-lived, strong even among large firms, and subsumes both the value and profitability premia. Weber further showed that after controlling for duration, the value and profitability premia disappear—suggesting that duration is a more fundamental characteristic than the traditional value factor. This finding reinforces RAPP’s theoretical grounding: the payback horizon N^* is functionally an inverse duration measure, and the pattern that shorter horizons (higher risk) correspond to higher required returns is precisely what the equity duration literature predicts.

Da and Warachka (2009) demonstrated that cash flow volatility is linked to equity duration and stock returns, providing additional evidence that the timing structure of expected cash flows is a first-order determinant of equity risk. Their work supports RAPP’s use of the earnings-weighted average recovery time (Payback Duration) as a risk metric.

Penman (2013) developed a financial statement analysis framework in which valuation is grounded in forecasting earnings over finite horizons rather than estimating terminal values. His emphasis on finite-horizon earnings accumulation—and skepticism toward terminal value dominance in DCF—aligns closely with RAPP’s philosophical approach, though Penman does not derive the specific DRPM calibration that makes the undiscounted sum exactly equal to DCF.

Dechow, Sloan, and Zha (2014) provided a comprehensive review of the relationship between stock prices and accounting earnings, documenting that earnings remain the single most important determinant of stock prices. Their survey of the accrual accounting literature supports RAPP’s use of EPS as the default cash flow metric: over RAPP-length horizons, accrual earnings and cash flows converge for most businesses (Dechow, 1994; Sloan, 1996).

RAPP’s Payback Duration $D_{RAPP} = \sum(t \cdot EPS_t) / \sum(EPS_t)$ is analytically related to Dechow, Sloan, and Soliman’s (2004) implied equity duration, but differs in three respects: (a) it is computed from analyst projections rather than realized financial statements, (b) it uses a finite horizon N^* rather than an infinite series, and (c) it is embedded within a complete valuation framework rather than serving as a standalone risk measure. Section 3.5 develops these connections formally.

2.4 Payback Period in Equity Valuation

The P/E ratio is, implicitly, a payback period: a P/E of 20 says that at current earnings, it takes 20 years to “earn back” the stock price. But P/E ignores growth (Damodaran, 2012). The PEG ratio attempts correction but lacks theoretical basis (Easton, 2004). Liu (2011) showed that a 10% required return maps to roughly a 10-year required payback, forming part of the foundation for both RAPP and the Potential Payback Period (PPP) framework.

Sam (2025a) proposed the PPP framework, deriving $PPP = \ln(r/(r-g))/\ln(1+g)$ —numerically identical to the DRPM formula derived independently in this paper. Both solve the same condition: $\sum(t=1 \text{ to } N) E_0(1+g)^t = E_0(1+g)/(r-g)$. The critical difference is in application. PPP uses N as a relative ranking score: a stock with $PPP = 8$ is “better” than one with $PPP = 12$ because earnings recover the price faster. RAPP uses N^* as a valuation instrument, proving (Appendix B) that the undiscounted sum through N^* years equals DCF fair value exactly. This theorem—that discounting is unnecessary when the horizon is correctly calibrated—is what transforms a ranking heuristic into a standalone valuation method. Without it, one has a useful screening tool but no theoretical license to omit discount factors from the valuation sum.

Sam’s additional metrics—the SIRR (Stock Internal Rate of Return) and the Hidden Value Zone—

serve different purposes from RAPP’s derivative metrics. SIRR measures the return implied by the current price and earnings trajectory, while RAPP’s Implied Payback Period translates price into risk perception. The Hidden Value Zone identifies stocks where PPP suggests undervaluation, while RAPP’s Payback Duration measures the sensitivity of valuation to late-period assumptions. The frameworks are complementary: PPP provides efficient screening across large universes, while RAPP provides the deep valuation analysis that follows screening.

Table 1: RAPP vs. PPP Methodological Comparison

Dimension	PPP (Sam 2025a-f)	RAPP (This Paper)
Core Philosophy	Relative ranking via payback score	Absolute valuation: discounting unnecessary at N^*
Formula	$N^{(PPP)} = \ln(r/(r-g))/\ln(1+g)$	$N^* = \ln(r/(r-g))/\ln(1+g)$
Key theorem	None (no fair-value proof)	$\sum_{t=1}^{N^*} EPS_t = V_{DCF}$ exactly
Output	Relative score (years)	Absolute price target (\$)
Unique metrics	SIRR, Hidden Value Zone	Payback Duration, Implied Payback Period

2.5 Valuation of High-Growth Companies

The valuation of high-growth companies poses particular challenges for DCF, as terminal value typically dominates total value and small changes in growth assumptions produce large valuation swings. Damodaran (2009) documented that traditional DCF models struggle with companies exhibiting negative current earnings, uncertain paths to profitability, and rapid structural change. The practice of “normalizing” earnings or using revenue multiples represents an implicit concession that the DCF architecture is poorly suited to these firms. RAPP’s transparent handling of scenario analysis—where adding or subtracting a year of earnings immediately reveals the valuation impact—addresses precisely this class of firms, where the debate is inherently about how long growth can be sustained rather than what discount rate should be applied.

2.6 Research Gap

Can payback logic be adapted to equity valuation with theoretical consistency? Under what conditions do undiscounted earnings sums equal DCF? How should risk be incorporated, and can the payback period be computed analytically from DCF inputs? How does the payback horizon relate to the equity duration metrics that have been shown to predict cross-sectional returns? This paper addresses each of these questions while connecting the framework to the established equity duration literature and distinguishing the approach from contemporaneous work using similar formulas for different purposes.

3. Theoretical Framework

3.1 The RAPP Model: Core Concept

At its core, RAPP asks a simple question: how many years of per-share cash flow does it take to justify the current stock price? The answer is the fair value. Formally:

$$P_0 = \sum_{t=1}^{N(r)} CF_t$$

where P_0 is fair price, CF_t is per-share cash flow in year t , and $N(r)$ is the acceptable payback period as a function of risk. The DRPM formula (Section 3.4) computes N directly from the required return r and growth rate g ; for typical equities, this produces horizons in the range of roughly 7–15 years, with higher-risk firms mapping to shorter horizons and lower-risk firms to longer ones.

The choice of cash flow metric—earnings per share versus free cash flow per share—is a deliberate design decision. The theoretically clean answer favors free cash flow, since what ultimately belongs to equity holders is cash, not accounting earnings. But there is a practical problem: FCF data is messier, less standardized, and harder to project than EPS. Over the horizon lengths that DRPM produces for typical equities, the cumulative difference between EPS and FCF per share washes out for most businesses. Working capital swings net to roughly zero over a full cycle, and unless a company is perpetually pouring capital into depreciating assets faster than those assets generate accounting income, the two numbers converge (Dechow, 1994; Sloan, 1996).

The framework therefore adopts a two-tier approach. The default implementation uses EPS:

$$P_0 = \sum_{t=1}^{N(r)} EPS_t \text{ (EPS-RAPP: default for most sectors)}$$

This applies to software companies, consumer brands, healthcare, financials—anything where the business is not defined by massive physical capital spending. Consensus EPS is widely available, analysts already think in P/E multiples, and the data is clean. For businesses where the gap between reported earnings and actual cash generation is persistent and structural—utilities reinvesting heavily in grid infrastructure, airlines constantly replacing aircraft—the analyst switches to:

$$P_0 = \sum_{t=1}^{N(r)} FCFPS_t \text{ (FCF-RAPP: for capital-intensive sectors)}$$

A concrete switching rule appears in Section 6.3: when the trailing five-year average EPS-to-FCF ratio drops below 0.7, use FCF-RAPP. This threshold captures approximately 15% of the S&P 500—mostly industrials, utilities, and transportation—leaving EPS-RAPP as the workhorse for the remainder.

One clarification deserves emphasis: RAPP is not the same as the traditional capital budgeting payback period. The textbook version takes the investment as given and asks “how long to recover it?” RAPP inverts this logic. It fixes the time horizon first—based on how risky the business is—and then asks “what does the cumulative cash flow over that horizon add up to?” That sum is the fair price. The criticism that payback ignores post-payback cash flows, while valid for capital budgeting, does not apply to RAPP: the post-payback earnings are implicitly captured through the DRPM calibration, which sets N^* at precisely the horizon where the undiscounted sum equals the full DCF value including terminal value (Section 3.4.1).

3.2 Theoretical Justification: Convergence to DCF

The non-obvious claim is that undiscounted per-share cash flows over a finite window produce a number close to DCF’s discounted-everything-plus-terminal-value approach. The reason is that two large effects push in opposite directions: discounting pulls future cash flow values down, while terminal value pushes them back up. Over a wide range of parameters, these forces cancel.

To be precise, consider a firm with constant growth g and required return r . The DCF value is:

$$V_{DCF} = E_0 \cdot \sum_{t=1}^T (1+g)^t / (1+r)^t + E_0(1+g)^{(T+1)} / ((r-g)(1+r)^T)$$

The RAPP value is simply the undiscounted cumulative sum:

$$V_RAPP(N) = E_0 \cdot (1+g)[(1+g)^N - 1] / g$$

Here E_0 represents the base-year per-share cash flow—EPS in most cases, or FCFPS for capital-heavy businesses (see Section 3.1 for the switching logic). The convergence proof does not depend on which metric is used; the mathematical structure is identical either way.

With $E_0 = \$10$, $r = 10\%$, and $g = 3\%$, the Gordon Growth Model gives $V_DCF = \$147.14$. RAPP with $N = 12$ years (close to $N^* = 12.1$) gives $V_RAPP = \$146.18$ —a 0.7% difference. For a two-phase model with $g_1 = 8\%$ for 5 years declining to $g_2 = 3\%$, the same horizon produces 1.3% divergence. This convergence holds broadly for established companies with moderate growth profiles. For high-growth firms, longer N is needed because earnings are backend-loaded. For very mature firms with disproportionately large terminal values, shorter N works better. A full proof appears in Appendix B.

3.3 Risk Adjustment Mechanism

RAPP adjusts for risk through time horizons rather than discount rates. Five perspectives support this approach.

Behavioral intuition. The golden-egg goose thought experiment (Section 1.1.1) provides illustrative evidence that when risk is introduced, finance professionals in this sample lowered their price—translating uncertainty directly into a shorter implied payback horizon. The risk-adjusted valuations imply 8–13 years of current earnings, consistent with CAPM-implied payback horizons. This pattern is consistent with payback-based thinking as a cognitive heuristic for assessing value under uncertainty (Kahneman, 2011; Simon, 1955). RAPP formalizes this intuition, and the DRPM formula extends it to the full range of equities with varying growth and risk profiles.

Mathematical grounding. Liu (2011) showed that a required return of r corresponds to a payback period of roughly $1/r$ years under steady-state conditions with low growth. An 8% required return gives $1/0.08 \approx 12.5$ years; 10% gives ≈ 10 years; 13% gives ≈ 7.7 years. The exact DRPM formula produces longer horizons when $g > 0$: at $g = 3\%$, an 8% required return gives $N^* = 15.9$ rather than 12.5. The $1/r$ approximation is most accurate for low-growth, steady-state firms; see the calibration table in Section 3.4.2 for exact values.

Survival probability interpretation. A risky firm has a higher probability of hitting distress, getting disrupted, or facing structural shifts that invalidate the earnings trajectory. If firm survival is modeled as an exponential process with hazard rate λ , the effective horizon where 90% of cumulative survival probability is captured is $N_effective = -\ln(0.1)/\lambda$. Higher λ means shorter horizons, consistent with RAPP.

Real options theory. In high-uncertainty environments, there is value in preserving flexibility—not committing too much capital to any single bet (Dixit & Pindyck, 1994; Trigeorgis, 1996). Requiring faster payback effectively prices in the option value of waiting. Investors demanding 8-year payback are implicitly saying they want their capital back quickly for redeployment if better opportunities arise.

Equity duration. Lettau and Wachter (2007) demonstrated that assets with shorter cash-flow duration earn higher expected returns in equilibrium. RAPP’s risk adjustment mechanism—assigning shorter horizons to riskier firms—is the direct valuation-framework analog of this asset pricing result: higher required returns map to shorter N^* through the DRPM formula, producing valuations whose cash-flow duration is inversely related to risk, exactly as equilibrium models predict.

The convergence of these five perspectives—behavioral evidence, mathematical necessity, survival probability, real options logic, and equilibrium asset pricing theory—provides robust support for the time-horizon approach to risk adjustment.

3.4 The Discount Rate-Payback Period Mapping (DRPM)

The DRPM is the most important theoretical addition in this paper. The original RAPP framework classified risk qualitatively: high, moderate, low. While practical, this invites the criticism that the horizon assignment is arbitrary. This section demonstrates that the payback period can be derived from a closed-form formula that takes the same inputs as DCF—eliminating arbitrariness while preserving the intuitive appeal of payback-based reasoning.

3.4.1 Derivation The DRPM formula can be derived entirely from first principles, requiring only the definition of discounted cash flow and two applications of the geometric series. The derivation proceeds in three steps: (1) DCF as an infinite geometric series yields the Gordon Growth Model, (2) the payback condition produces a finite geometric series, and (3) solving for the horizon yields the DRPM formula. Presenting the complete chain—from the DCF axiom through to the closed-form N^* —establishes that RAPP rests on no assumptions beyond those already embedded in standard DCF theory.

Step 1: From DCF to the Gordon Growth Model. Consider a firm with base-year earnings E_0 and constant earnings growth rate g . The year- t earnings are $E_0(1+g)^t$, and the DCF fair value is the sum of all future earnings discounted at required return r :

$$P = E_0(1+g)/(1+r) + E_0(1+g)^2/(1+r)^2 + E_0(1+g)^3/(1+r)^3 + \dots$$

This is an infinite geometric series with first term $a = E_0(1+g)/(1+r)$ and common ratio $q = (1+g)/(1+r)$. Provided $g < r$, the ratio satisfies $|q| < 1$, and the series converges:

$$P = a / (1 - q) = [E_0(1+g)/(1+r)] / [1 - (1+g)/(1+r)]$$

The denominator simplifies as follows: $1 - (1+g)/(1+r) = [(1+r) - (1+g)]/(1+r) = (r - g)/(1+r)$. Substituting:

$$P = [E_0(1+g)/(1+r)] \times [(1+r)/(r - g)] = E_0(1+g) / (r - g)$$

This is the Gordon Growth Model (Gordon, 1959). The condition $g < r$ is not merely a mathematical convenience but a valuation existence condition: if earnings grow faster than the required return in perpetuity, the asset's present value is infinite under any framework.

Step 2: The payback condition. RAPP asks: at the Gordon Growth fair price P , how many years N^* of cumulative (undiscounted) earnings does it take to recover the purchase price? The cumulative earnings over N^* years form a finite geometric series:

$$S(N) = E_0(1+g) + E_0(1+g)^2 + \dots + E_0(1+g)^{N^*} = E_0(1+g) \cdot [(1+g)^{N^*} - 1] / g$$

Setting $S(N^*) = P$ and substituting the Gordon Growth expression for P :

$$E_0(1+g) \cdot [(1+g)^{N^*} - 1] / g = E_0(1+g) / (r - g)$$

Step 3: Solving for N^* . Cancel $E_0(1+g)$ from both sides:

$$[(1+g)^{N^*} - 1] / g = 1 / (r - g)$$

Multiply both sides by g :

$$(1+g)^{N^*} - 1 = g / (r - g)$$

Add 1 to both sides and consolidate:

$$(1+g)^{N^*} = 1 + g/(r - g) = (r - g + g) / (r - g) = r / (r - g)$$

Take the natural logarithm of both sides:

$$N^* \cdot \ln(1+g) = \ln(r / (r - g))$$

Divide by $\ln(1+g)$:

$$N^* = \ln(r/(r-g)) / \ln(1+g)$$

where N^* is the optimal payback horizon (years), r is the required return (from CAPM: $r = r_f + \beta \cdot \text{ERP}$), and g is the long-run earnings growth rate. Higher r (more risk) shortens N ; *higher g (more growth) extends it*. N is independent of the current earnings level E_0 —the formula works the same regardless of company size.

The key implication—discounting is unnecessary at N^* : This derivation establishes more than a formula. It proves that an analyst who knows N^* can obtain the correct DCF fair value by summing undiscounted earnings over N^* years—no discount factors, no terminal value calculation, no WACC construction. The result is exact, not approximate.

The mechanism is that DCF applies two adjustments to raw earnings: discounting (which lowers the value of future earnings) and a terminal value (which adds back the present value of all post-forecast earnings). At the horizon N , *these two adjustments cancel exactly—the amount subtracted by discounting equals the amount added by terminal value. The net effect on the sum is zero. This is not a coincidence or an approximation: it is a mathematical identity that follows directly from the DRPM definition of N .*

To understand this intuitively, consider that discounting penalizes distant earnings more heavily, while terminal value rewards the investor for earnings that continue beyond the forecast horizon. When the horizon is too short, the terminal value dominates and the investor is essentially betting on unobservable future cash flows. When the horizon is too long, the discounting dominates and the investor is unnecessarily penalizing earnings that are actually captured in the projection. N^* is the horizon where these two forces are in perfect equilibrium—the “Goldilocks” horizon where neither effect dominates.

Formally, for any $N \neq N^*$, *the undiscounted sum diverges from V_{DCF} . For $N < N^*$, RAPP underestimates; for $N > N^*$, RAPP overestimates. Only at N^* is the equivalence exact.* This is proven rigorously in Appendix B.

For small g , the formula simplifies via Taylor expansion to $N^* \approx 1/r$ —Liu’s (2011) result. At $r = 10\%$, this gives $N^* \approx 10$ years. A more useful form expresses N^* in terms of the spread $s = r - g$: $N^* = \ln(1+g/s)/\ln(1+g)$, showing that payback is driven primarily by the gap between required return and growth rate. This “spread form” provides useful intuition: N^* depends most strongly on the spread rather than on r or g individually. Narrowing the spread—whether by raising g or lowering r —extends the payback horizon, and the extension accelerates as s shrinks (consistent with the singularity behavior explored in Section 3.4.6). Note, however, that N^* is not invariant across (r, g) pairs with the same spread: at $s = 5\%$, a firm with $r = 10\%$ and $g = 5\%$ has $N^* \approx 14.2$ years, while a firm with $r = 15\%$ and $g = 10\%$ has $N^* \approx 11.5$ years. The higher-growth firm reaches payback faster because each year’s earnings increment is proportionally larger.

The economic interpretation is that N^* answers the question: “At what point has the company generated enough cumulative earnings to compensate the investor for the risk of holding the stock?” The DRPM formula converts this question from a vague judgment call into a precise function of observable parameters.

Note on mathematical identity with PPP: The DRPM formula is numerically identical to Sam’s (2025a) PPP formula. The difference is in application: PPP uses N as a relative ranking score; RAPP uses N^* as the valuation horizon licensing the omission of discount factors. Without the convergence proof (Appendix B), one would have no theoretical justification for omitting discounting. With it, the omission is a mathematically exact substitution.

3.4.2 Calibration The DRPM formula can be inverted: $r = g + g/((1+g)^{N^*} - 1)$. This inverted form is more intuitive: the analyst observes g from the earnings model and N reflecting risk judgment, then checks whether the implied r aligns with CAPM.

Table 3A: Implied Required Return r by Payback Period N and Growth Rate g

N (years)	1% (g)	2% (g)	3% (g)	5% (g)	7% (g)
8	13.1%	13.7%	14.2%	15.5%	16.7%
10	10.6%	11.1%	11.7%	13.0%	14.2%
12	8.9%	9.5%	10.0%	11.3%	12.7%
15	7.2%	7.8%	8.4%	9.6%	11.1%

The ~10-year row at $g = 3\text{--}5\%$ implies $r = 11\text{--}13\%$, squarely in the range of typical equity required returns, confirming that the 10-year anchor reflects real market risk pricing.

3.4.3 Connecting to CAPM Substituting $r = r_f + \beta \cdot \text{ERP}$ into the DRPM formula yields N^* as a function of four observable parameters: the risk-free rate, the equity risk premium, the company’s beta, and the long-run growth rate. This CAPM-DRPM formula is the formal bridge between DCF and RAPP. It establishes that every DCF valuation implicitly assumes a specific payback horizon, and every RAPP payback horizon implicitly embeds a specific required return—the two frameworks are different representations of the same underlying economic relationship.

DRPM does not eliminate discretion entirely—it relocates it. In DCF, the subjective judgment is the discount rate (or more precisely, the combination of risk-free rate, equity risk premium, and beta that produces it). In RAPP, the subjective judgment is the terminal-state beta and long-run growth rate. The advantage of the latter is twofold: terminal beta is arguably more tractable to estimate than a spot WACC (one asks “will this company mature to an average-risk profile?” rather than “what is the correct cost of equity to five decimal places?”), and the growth rate is already required for DCF’s terminal value anyway. RAPP makes both inputs transparent and their valuation consequences immediate.

3.4.4 Multi-Phase Growth and the Terminal Growth Rate Real companies do not grow at constant rates. For DRPM calibration, the analyst must use the long-term stable growth rate g_2 , since $g_1 > r$ (common for high-growth firms) makes N^* undefined. The high-growth phase is captured through year-by-year EPS projections in the RAPP sum, not through the DRPM calibration. This

two-level structure—explicit year-by-year projections for the high-growth phase, DRPM calibration using only the terminal growth rate—is the practical resolution of multi-phase growth within RAPP.

Why RAPP does not require an external ceiling on g . In DCF, the terminal growth rate must be constrained below the discount rate because the Gordon Growth denominator $1/(r-g)$ explodes as $g \rightarrow r$. This structural fragility forces practitioners to impose external ceilings (typically $g \leq$ nominal GDP growth; see Damodaran, 2012). RAPP faces no such structural necessity. The DRPM formula maps g to a logarithmic output: as g rises toward r , N^* increases smoothly and without bound, but the output always retains economic meaning. A firm with $g = 9\%$ and $r = 10.5\%$ produces $N^* = 22.6$ years and fair value = \$727 (for $E_0 = \$10$)—a long payback, but an interpretable one. The same inputs in DCF produce a terminal value of $66.7 \times$ FCF, which conveys no actionable information.

The only hard constraint RAPP requires is $g < r$. This is not a modeling assumption but a valuation existence condition: if a firm’s earnings grow faster than the investor’s required return in perpetuity, the asset’s value is infinite under any framework—DCF, RAPP, or otherwise. For any $g < r$, RAPP produces a finite, interpretable N^* and a computable fair value.

RAPP’s built-in sanity check. The logarithmic structure of DRPM provides a natural self-correcting mechanism that substitutes for external growth ceilings. When an analyst inputs an aggressive g , the result is not a model blowup but a transparently long payback period. Compare:

- $g = 5\%$, $r = 10.5\%$: $N^* = 13.3$ years, fair value = \$191
- $g = 8\%$, $r = 10.5\%$: $N^* = 18.6$ years, fair value = \$432
- $g = 9\%$, $r = 10.5\%$: $N^* = 22.6$ years, fair value = \$727

An investment committee told “this valuation requires a 23-year payback” will naturally question whether the growth assumption is sustainable. The payback period itself functions as a ceiling—not imposed externally by the model, but evaluated internally by the analyst’s judgment. This is a structural advantage over DCF, where the equivalent question (“is a $66.7 \times$ terminal multiple reasonable?”) generates confusion rather than insight.

Practical guidance: The Sector Excess Growth Rate (α). While RAPP does not theoretically require a growth ceiling, practitioners benefit from a systematic framework for estimating g_{terminal} . The sector excess growth rate α provides this framework by anchoring g_{terminal} to observable historical data:

$$g_{\text{terminal}} = g_{\text{nominal_GDP}} + \alpha$$

where $g_{\text{nominal_GDP}} \approx 4\text{--}5\%$ (US baseline: real 2–3% + inflation 2%) and α represents the sector’s historical long-run growth premium over the broader economy. This is not a constraint imposed on the model but a calibration tool that helps the analyst form a defensible g estimate.

Estimating α proceeds from historical data. The analyst computes the sector’s 20–30 year compound annual growth rate and subtracts the corresponding period’s nominal GDP growth. For example, global semiconductor revenue grew at approximately 8% annually from 1990–2020, against ~5% nominal GDP growth, yielding $\alpha \approx 3\%$. Enterprise software shows $\alpha \approx 2\text{--}3\%$; traditional industrials show $\alpha \approx 0\text{--}1\%$. These estimates are necessarily backward-looking, but for mature sectors with established competitive structures, historical α provides a defensible anchor. For the majority of equities—consumer staples, financials, healthcare, industrials— α is near zero, and $g_{\text{terminal}} \approx g_{\text{nominal_GDP}}$ is the natural default.

Table 3C- α : Illustrative Sector Growth Rates and α

Sector	Historical CAGR (20yr)	Nominal GDP	α	Typical g_terminal
Semiconductors	~8%	~5%	~3%	~8%
Enterprise Software	~7-8%	~5%	~2-3%	~7-8%
Healthcare Services	~6-7%	~5%	~1-2%	~6-7%
Consumer Staples	~5%	~5%	~0%	~5%
Traditional Industrials	~4-5%	~5%	~0%	~4-5%

For the default case—the majority of equities where no clear sector tailwind exists—the analyst simply uses $g_{\text{terminal}} \approx g_{\text{nominal_GDP}}$ (4-5%). This produces N^* in the 10-15 year range for typical betas, squarely within the zone where RAPP’s convergence with DCF is tightest (Appendix B).

The sensitivity of N^* to α is material, particularly for firms in structurally growing sectors:

Table 3C- α 2: N^* Sensitivity to g_{terminal} ($r = 10.5\%$, $g_{\text{GDP}} = 5\%$)

α	g_{terminal}	N^*	ΔN^* vs $\alpha=0$
0%	5.0%	13.3 yr	—
1%	6.0%	14.5 yr	+1.2 yr
2%	7.0%	16.2 yr	+2.9 yr
3%	8.0%	18.6 yr	+5.3 yr

An analyst who assigns $\alpha = 3\%$ rather than $\alpha = 0\%$ is implicitly extending the valuation horizon by over five years. Whether this extension is justified is a business judgment, not a model parameter—and RAPP makes the consequence of that judgment immediately visible in the payback period.

Conglomerate Treatment. Diversified companies operating across multiple sectors present a distinct calibration challenge, since no single sector α applies. Three approaches are available, in decreasing order of precision:

The preferred approach is a *weighted-average* α , computed as $\alpha_{\text{firm}} = \sum_i (w_i \cdot \alpha_i)$, where w_i is the profit contribution weight of business segment i and α_i is the sector excess growth rate for that segment. For Samsung Electronics, one might assign $\alpha_{\text{semiconductor}} \approx 3\%$ (60% of operating profit) and $\alpha_{\text{consumer_electronics}} \approx 0-1\%$ (40%), yielding $\alpha_{\text{firm}} \approx 2\%$. This approach preserves segment-level information while producing a single g_{terminal} for DRPM calibration.

A simplified alternative uses the *dominant segment approach*: when one business line contributes >60% of operating profit, its sector α serves as the firm-level α . This is appropriate for conglomerates with a clear profit center—Alphabet (advertising), Amazon (AWS + retail)—where the secondary segments have minimal impact on the terminal growth rate.

For complex conglomerates where no segment dominates, *sum-of-the-parts RAPP* computes N^* and fair value separately for each segment, then aggregates using profit weights. This is the most precise approach but requires segment-level financial projections, making it impractical for routine screening.

Nascent Industries Without Historical α . The α framework assumes a sector with sufficient history—at least 15–20 years of revenue data—to estimate long-run excess growth. For genuinely nascent industries—humanoid robotics, space commerce, quantum computing, brain-computer interfaces—no such history exists. The TAM itself may be starting from near zero, the competitive structure is undefined, and the range of plausible long-run outcomes spans from commodity failure to transformative platform dominance. Assigning a single α to such sectors would embed false precision into a fundamentally uncertain parameter.

For these industries, the recommended approach replaces point-estimate α with explicit scenario analysis on g_{terminal} :

Table 3C-N: Scenario-Based g_{terminal} for Nascent Industries ($g_{\text{GDP}} = 5\%$)

Scenario	g_{terminal}	α equivalent	Assumption	N^* ($\beta=1.2$)
Bull	$g_{\text{GDP}} + 3\% = 8\%$	3%	Achieves semiconductor-scale structural growth; the industry becomes a foundational economic layer	14.7 yr
Base	$g_{\text{GDP}} + 1\% = 6\%$	1%	Moderate structural growth; the industry matures into a stable but growing sector	12.2 yr
Bear	$g_{\text{GDP}} + 0\% = 5\%$	0%	GDP convergence; the industry commoditizes or fails to achieve structural differentiation	11.3 yr

Each scenario produces a distinct N^* and fair value. The analyst then forms an expected value by assigning probability weights to each scenario:

$$V_{\text{expected}} = w_{\text{bull}} \cdot V_{\text{bull}} + w_{\text{base}} \cdot V_{\text{base}} + w_{\text{bear}} \cdot V_{\text{bear}}$$

For example, a humanoid robotics firm with $E_0 = \$2$, $\beta = 1.2$: the Bull case ($N^* = 14.7$ years) yields $V_{\text{bull}} = \$56.84$; the Base case ($N^* = 12.2$ years) yields $V_{\text{base}} = \$36.55$; the Bear case ($N^* = 11.3$

years) yields $V_{\text{bear}} = \$30.88$. An analyst assigning 20%/50%/30% weights computes $V_{\text{expected}} = 0.2 \times \$56.84 + 0.5 \times \$36.55 + 0.3 \times \$30.88 = \$38.91$.

This approach has three advantages over DCF scenario analysis for nascent industries. First, the scenarios are expressed in interpretable units: “this stock is a 15-year payback under the bull case, 11-year under the bear case.” An investment committee can debate whether a humanoid robotics firm deserves a 15-year payback; they cannot productively debate whether the terminal EBITDA multiple should be 25× or 40×. Second, the scenario spread—here \$31 to \$57, or roughly 1.8×—makes the uncertainty visible rather than hiding it inside a single terminal value. Third, the probability weights are explicit and adjustable: as the industry matures and the competitive structure clarifies, the analyst shifts weight from bear to base or from base to bull, producing a transparent and auditable valuation evolution over time.

The scenario framework also provides a natural on-ramp to the standard α framework. As the nascent industry accumulates 10-15 years of revenue history, the analyst can compute a preliminary historical α and transition from scenario-weighted valuation to a point-estimate α , reducing complexity as uncertainty resolves. The transition criterion is straightforward: when the industry’s trailing 10-year CAGR stabilizes within a 2-percentage-point band, the analyst can adopt the midpoint as α and retire the scenario framework for that sector.

Summary: g_{terminal} estimation by company type. The framework provides a clear decision tree:

- *Most equities* (consumer, financial, industrial, healthcare): $g_{\text{terminal}} \approx g_{\text{GDP}}$ (4-5%). No α estimation required.
- *Structurally growing sectors* (semiconductors, enterprise software): $g_{\text{terminal}} = g_{\text{GDP}} + \alpha$, where α is estimated from 20+ years of sector history.
- *Conglomerates*: Weighted-average α across business segments, or dominant-segment α when one unit contributes >60% of profits.
- *Nascent industries* (humanoid robotics, quantum computing): Bull/Base/Bear scenario analysis on g_{terminal} with probability-weighted expected value.

In all cases, the analyst’s g assumption flows through DRPM to produce an N^* that is immediately interpretable. If N^* exceeds 20 years, the assumption deserves scrutiny—not because the model requires it, but because few businesses sustain above-GDP growth for two decades.

Table 3C: DRPM Horizon by Beta and Terminal Growth Rate ($r_f = 4.0\%$, $\text{ERP} = 6.5\%$)

β	r	N^* ($g=3\%$)	N^* ($g=5\%$)	N^* ($g=6\%$)	N^* ($g=8\%$, $\alpha=3\%$)
0.6	7.9%	16.2	20.5	24.5	—
0.8	9.2%	13.4	16.1	18.1	26.5
1.0	10.5%	11.4	13.3	14.5	18.6
1.2	11.8%	9.9	11.3	12.2	14.7
1.5	13.75%	8.3	9.3	9.8	11.3
2.0	17.0%	6.6	7.1	7.5	8.3

*The $g = 8\%$ column applies only to high- α sectors (e.g., semiconductors) where $\alpha \approx 3\%$ is historically justified. For $\beta = 0.6$, $g = 8\%$ approaches r , making N extremely large and economically implausible.**

The terminal growth rate is the single most consequential input in RAPP. A generic stock with $E_0 = \$10$, $r = 10.5\%$: $g = 5\%$ gives $N^* = 13.3$ years and fair value = \$191; $g = 8\%$ gives $N^* = 18.6$ years and fair value = \$432; $g = 9\%$ gives $N^* = 22.6$ years and fair value = \$727. RAPP makes the consequence of each growth assumption immediately transparent—the analyst can see exactly how many additional years of payback each percentage point of growth requires, and judge whether the implied wait is justified.

3.4.5 The Mathematical Boundary: When N^* Diverges As $r - g$ narrows, N^* increases without bound. When $g \geq r$, the formula is undefined. This is the same singularity that causes the Gordon Growth Model to blow up. Unlike DCF, where this singularity produces uninterpretable valuations, RAPP translates it into progressively longer—but always finite and meaningful—payback horizons. The only hard constraint is $g < r$; for any g satisfying this condition, RAPP produces a valid output.

Companies where N^* exceeds 20 years—regulated utilities, toll roads, essential infrastructure—sit at RAPP’s natural boundary. Their DCF values are similarly dominated by terminal value, and in both frameworks, the analyst is effectively betting on very long-duration cash flows. Rather than capping N^* artificially, RAPP acknowledges this as an honest boundary: the analyst should recognize heightened sensitivity to the growth assumption and may wish to supplement with income-based valuation approaches.

An optional tail premium factor can approximate RAPP for these edge cases: $P_0^{\wedge}(\text{adjusted}) = \sum_{t=1}^{N_{\text{work}}} CF_t \cdot (1 + \alpha)$, where $\alpha = (N^* - N_{\text{work}})/N^*$ and $N_{\text{work}} = \min(N, 20)$. *This is a pragmatic approximation, not a theoretical extension; the analyst trades exact convergence for tractability when N exceeds the horizon over which detailed projections are credible.*

In practice, the typical range of g_{terminal} (4–8% for most actively analyzed equities) and the minimum plausible beta ($\beta \approx 0.6\text{--}0.8$) together produce a “practical zone” for N^* that encompasses the vast majority of valuation problems: approximately 7–20 years for equities with $\beta \in [0.7, 2.0]$ and $g \in [2\%, 8\%]$. N^* values outside this range are not model errors—they are signals that the growth or risk assumption deserves additional scrutiny.

3.4.6 Structural Resolution of the Terminal Value Singularity RAPP’s DRPM formula responds to the $r\text{-}g$ singularity through a logarithmic function rather than DCF’s hyperbolic one. DCF’s sensitivity $\partial(TV/FCF)/\partial g = 1/(r-g)^2$ grows as the square of the inverse spread; RAPP’s dominant sensitivity term grows as the first power. The ratio between DCF and RAPP sensitivity increases monotonically as $g \rightarrow r$.

Table 3D: Terminal Value Singularity — DCF vs. RAPP ($r = 10\%$)

g	$r-g$	DCF: TV/FCF	RAPP: N^*	Sensitivity Ratio
3%	7.0%	14.3×	12.1 yr	2.3×
7%	3.0%	33.3×	17.8 yr	4.5×
9%	1.0%	100.0×	26.7 yr	11.4×
9.5%	0.5%	200.0×	33.0 yr	21.4×
9.9%	0.1%	1,000.0×	48.8 yr	—

Sensitivity Ratio is defined as $(\partial(TV/FCF)/\partial g) / (\partial N^/\partial g)$, evaluated at each (r, g) point. It measures how much faster DCF’s terminal value responds to a marginal change in g compared to RAPP’s*

payback horizon. The last row is omitted because $\partial N^*/\partial g \rightarrow \infty$ in the same limit as $\partial(TV/FCF)/\partial g$, making the ratio analytically indeterminate at $r-g = 0.1\%$.

The divergence in interpretability grows monotonically. At moderate g , both frameworks produce reasonable outputs. As g approaches r , DCF's output ceases to be interpretable (200× or 1,000× terminal value multiples), while RAPP's output always retains economic meaning ("33 years to recover your investment"). Existing remedies—the Value Driver Formula (Koller et al., 2020), fade models (Holland, 2018), the H-Model (Fuller & Hsia, 1984)—all operate within the $1/(r-g)$ structure. RAPP escapes it entirely without introducing additional parameters.

The structural advantage can be stated concisely: DCF converts a narrow spread into an enormous price; RAPP converts it into a long wait. The first is fragile and uninterpretable; the second is robust and actionable. An analyst told that a company is worth "1,000× free cash flow" has no useful information for investment decision-making; an analyst told that the company requires "49 years to recover the investment" has a clear, interpretable signal: do not invest.

This reframing has important implications for how analysts communicate about high-growth, narrow-spread companies. Rather than presenting DCF models with terminal values that dominate the output and invite spurious precision debates, the analyst can present the RAPP horizon and let the investment committee decide whether the implied wait is acceptable given the company's competitive position and the analyst's conviction in long-term growth sustainability.

3.5 Payback Duration

Define Payback Duration as the earnings-weighted average year of recovery:

$$D_RAPP = \sum_{t=1}^N t \cdot EPS_t / \sum_{t=1}^N EPS_t$$

This metric is analytically related to the implied equity duration of Dechow, Sloan, and Soliman (2004), who demonstrated that equity duration—the weighted-average timing of a firm's expected cash flows—is positively correlated with stock price volatility and beta. RAPP's Payback Duration differs in three ways: it uses analyst projections rather than realized statements, operates over a finite DRPM-calibrated horizon rather than an infinite series, and is embedded within a complete valuation framework.

For constant growth, the closed form is: $D_RAPP = N(1+g)^N / ((1+g)^N - 1) - 1/g$. Under zero growth, $D_RAPP = (N+1)/2$ (the midpoint). Duration increases monotonically with growth rate, confirming that higher-growth firms have more backend-loaded recovery profiles. At $g = 5\%$ and $N^* = 10$, $D_RAPP = 5.90$ years (59% of horizon); at $g = 15\%$, it rises to 6.62 years (66%).

The elasticity of RAPP value with respect to growth is $\Delta V \approx (D_RAPP/(1+g)) \cdot \Delta g \cdot V$, directly analogous to modified bond duration. This connection to the duration literature (Lettau & Wachter, 2007; Weber, 2018; Gonçalves, 2021) provides theoretical support for using D_RAPP in portfolio construction: low-duration stocks ($D_RAPP < 5$) exhibit defensive characteristics, with valuation relatively insensitive to changes in growth expectations; high-duration stocks ($D_RAPP > 7$) exhibit offensive characteristics, with large valuation swings in response to growth surprises.

The portfolio construction application is straightforward. During periods of decelerating economic growth or rising interest rates, a portfolio tilted toward low-duration stocks should exhibit lower valuation volatility. During expansion phases, high-duration stocks offer greater upside from positive growth surprises. Weber (2018) and Gonçalves (2021) provided cross-sectional evidence consistent with this prediction, with short-duration stocks earning a significant premium over long-duration

stocks, and Gonçalves (2021) further showing that this premium is strong even among large firms and subsumes the value and profitability premia. The D_RAPP metric operationalizes this insight within a complete valuation framework rather than as a standalone factor. Full derivations appear in Appendix F.

3.6 Implied Payback Period and Market Beta Diagnostic

The Implied Payback Period, N_{implied} , is the RAPP analog of implied volatility: given the market price, what horizon N makes RAPP value equal to that price? Two implementations are possible, depending on which growth assumption is used:

(i) *Projection-based N_{implied}* : Using the analyst’s explicit year-by-year EPS (or FCFPS) projections, find the horizon N such that $\sum_{t=1}^N CF_t = P_0$. This answers “over how many years of the projected earnings stream does the cumulative sum reach the market price?”

(ii) *DRPM-based N_{implied} (constant- g approximation)*: Assume a single long-run growth rate g and invert the DRPM relation: solve $V_{\text{RAPP}}(N; E_0, g) = P_0$ to obtain N_{implied} , then back out $r_{\text{implied}} = g + g/((1+g)^{N_{\text{implied}}} - 1)$ and $\beta_{\text{implied}} = (r_{\text{implied}} - r_f)/ERP$. This answers “what constant-growth horizon, and hence what required return and beta, is the market implicitly pricing?”

The two definitions coincide when the analyst’s projection happens to follow a constant- g path, but diverge for multi-phase trajectories. Table 4 and the case-study values below use the DRPM-based form, because its purpose is to translate price into a risk-perception metric (β_{implied}). Section 5 presents projection-based N_{implied} alongside DRPM-based N_{implied} where the two differ materially.

Table 4: Market-Implied Beta by Payback Period (DRPM-based) ($r_f = 4.0\%$, $ERP = 6.5\%$, selected values)

N_{implied}	$g=3\%$: Impl. β	$g=5\%$: Impl. β	$g=10\%$: Impl. β
8	1.58	1.76	2.27
10	1.19	1.38	1.89
12	0.93	1.12	1.64
15	0.67	0.87	1.41

For the three case studies (January 2026 prices), DRPM-based diagnostics:

Company	Price	N_{implied} (DRPM)	g	Market β_{implied}	CAPM β
Carvana	\$400	10.1 yr	3%	1.19	1.2
Tesla	\$385	10.0 yr	4%	1.28	1.8
Palantir	\$88	11.8 yr	5.5%	1.19	1.5

For completeness, the projection-based N_{implied} using the Appendix A base-case EPS streams yields: Carvana ≈ 10.4 yr (vs. DRPM 10.1), Tesla ≈ 10.0 yr (vs. DRPM 10.0), Palantir ≈ 10.8 yr (vs. DRPM 11.8). Tesla’s two values coincide because its base-case trajectory is well approximated

by a single-g path over the relevant horizon; Palantir’s divergence reflects its backend-loaded projection (86% Payback Duration, Section 5.3), where the late-period earnings ramp cumulates faster than constant-g would predict. When the two measures disagree, the DRPM-based value is the correct input for β_{implied} (since β_{implied} is derived from the DRPM inversion), while the projection-based value is the more intuitive answer to “how many years of the actual forecast stream does the market’s price cover?”

Carvana’s market price embeds $\beta = 1.19$ —essentially the mature-state estimate, suggesting the market views Carvana as having successfully completed its transition from high-risk turnaround to established operator. Tesla’s β_{implied} of 1.28 is materially below its current CAPM β of 1.8, indicating the market prices Tesla as achieving near-mature risk profile—implying successful execution across all revenue segments. Palantir’s β_{implied} of 1.19 is similarly well below its CAPM β of 1.5, reflecting the market’s confidence in the AI platform growth trajectory. These diagnostics are actionable: an analyst who believes Tesla’s FSD/robotaxi segment carries substantially more execution risk than priced ($\beta > 1.5$) would see the market as complacent about autonomous driving timelines, while an analyst who believes Palantir’s AIP adoption will accelerate beyond the base case would see remaining upside.

3.6.1 Three Inversions for High-Growth Firms For firms in high- α sectors where the terminal growth assumption materially affects valuation, three additional inversions provide complementary market diagnostics. Each fixes a different variable and solves for the remaining unknown, reducing the degrees of freedom and enabling precise identification of where the analyst disagrees with the market.

Inversion 1: Implied α . Fix the payback horizon at the sector-average N^*_{sector} (e.g., 11 years for technology) and solve for the α that reconciles the market price with RAPP fair value. If $P_0 = \sum_{t=1}^{N^*_{\text{sector}}} CF_t$ under growth $g_{\text{GDP}} + \alpha_{\text{implied}}$, then α_{implied} reveals the market’s embedded assumption about the firm’s long-run sector outperformance. An α_{implied} of 4.5% for a semiconductor firm—when the historical sector α is 3%—signals that the market is pricing in structural outperformance beyond what historical data supports.

Inversion 2: Implied N^* (α fixed). Fix α at the sector’s historical value and solve for the payback horizon that equates cumulative earnings to the current price: $P_0 = \sum_{t=1}^{N^*_{\text{implied}}} CF_t$ where $g_{\text{terminal}} = g_{\text{GDP}} + \alpha_{\text{fixed}}$. This produces the most natural RAPP diagnostic: “the market is pricing this stock as a 16-year payback.” The analyst then judges whether 16 years is reasonable given the firm’s competitive position and risk profile. This inversion is particularly powerful because it collapses all market disagreement into a single, interpretable number—the implied wait time—while grounding the growth assumption in observable historical data rather than leaving it as a free parameter.

Inversion 3: Break-even α . Fix an upper bound on acceptable payback (e.g., $N^*_{\text{max}} = 15$ years, reflecting the practical limit of credible projections) and solve for the minimum α required to make the current price “fair” within that horizon: $P_0 = \sum_{t=1}^{N^*_{\text{max}}} CF_t$ under $g_{\text{GDP}} + \alpha_{\text{min}}$. If α_{min} exceeds 3%—the upper bound of historically observed sector premia—the stock requires assumptions outside the historical envelope to justify its price within a reasonable payback horizon. This is a strong overvaluation signal.

Table 4B: Three-Inversion Diagnostic Applied to Case Studies ($g_{\text{GDP}} = 5\%$)

Company	Inversion 1: α_{implied} ($N^*=11\text{yr}$)	Inversion 2: N^*_{implied} ($\alpha=\text{sector}$)	Inversion 3: α_{min} ($N^*_{\text{max}}=15\text{yr}$)
Carvana (CVNA, \$400)	$\alpha = 1.8\%$	$N^* = 10.1 \text{ yr}$ ($\alpha=0\%$)	$\alpha_{\text{min}} = 0\%$
Tesla (TSLA, \$385)	$\alpha = 1.5\%$	$N^* = 10.0 \text{ yr}$ ($\alpha=1\%$)	$\alpha_{\text{min}} = 0\%$
Palantir (PLTR, \$88)	$\alpha = 3.5\%$	$N^* = 11.8 \text{ yr}$ ($\alpha=3\%$)	$\alpha_{\text{min}} = 1.5\%$

Carvana and Tesla produce α_{implied} within the historical range, suggesting their prices are consistent with achievable sector growth assumptions. Palantir’s α_{implied} of 3.5% slightly exceeds the historical upper bound of ~3% for enterprise software, flagging that the market requires modestly above-historical sector outperformance to justify the current price—a quantitative expression of the “AI platform premium” thesis.

The three inversions are complementary and should be used together. Implied α reveals what the market believes about the firm’s sector; Implied N^* reveals how long the market is willing to wait; Break-even α reveals what assumptions are *required* to avoid overvaluation. Together, they transform the single-point RAPP valuation into a comprehensive market belief diagnostic.

3.6.2 RAPP-Implied P/E Under constant growth, RAPP provides a closed-form justified trailing P/E: $P_0/E_0 = (1+g)[(1+g)^{N^*} - 1]/g$. The corresponding forward P/E is $P_0/EP_{S1} = [(1+g)^{N^*} - 1]/g$. Since N^* is determined by β and g , the justified P/E becomes a pure function of these parameters.

Table 5: RAPP-Justified Trailing P/E (P_0/E_0) ($r_f = 4.0\%$, $ERP = 6.5\%$, selected values)

β	$g=2\%$	$g=3\%$	$g=5\%$	$g=8\%$
0.8	14.2x	16.6x	25.0x	90.0x
1.0	12.0x	13.7x	19.1x	43.2x
1.2	10.4x	11.7x	15.4x	28.4x
1.5	8.7x	9.6x	12.0x	18.8x
2.0	6.8x	7.4x	8.8x	12.0x

Growth has exponentially more impact than beta on justified P/E. For $\beta = 1.2$, moving from $g = 3\%$ to $g = 8\%$ raises justified trailing P/E from 11.7x to 28.4x. The S&P 500 trailing P/E of ~20x at $\beta \approx 1.0$ requires $g \approx 4\text{-}5\%$, precisely the long-run nominal earnings growth assumption most strategists use.

3.6.3 Implied Growth Rate The most powerful inversion runs the opposite direction: given the observed P/E and a beta estimate, what long-run growth rate must the market be assuming? This transcendental equation, solved numerically, makes the market’s embedded growth assumption explicit. Tesla at 85x forward P/E with $\beta = 1.5$ requires $g^* \approx 12.4\%$ sustained over 20 years; Palantir at 160x requires $g^* \approx 13.0\%$ over 24 years. These growth requirements—not the multiples themselves—are the actionable outputs for investment debate.

This inversion provides a concrete framework for fundamental analysis. An analyst evaluating Tesla can ask: “Is 12.4% annual EPS growth for 20 years achievable?” This decomposes into observable components: unit volume growth, margin expansion, revenue mix shift toward FSD and energy, and share count dynamics. Each component can be independently assessed against industry data and competitive dynamics. The implied growth rate test makes it explicit what the market requires and allows the analyst to determine whether those requirements are achievable, aggressive, or unrealistic. When the implied growth rate substantially exceeds the sector’s historical α -adjusted growth rate ($g_{GDP} + \alpha$, typically 5-8%), the market is implicitly assuming the company will sustain growth well beyond what comparable sectors have historically achieved—a signal that warrants scrutiny, though not necessarily a conclusion of overvaluation.

3.7 Comparison with Alternative Methods

RAPP versus DCF: With DRPM, formally equivalent—same inputs, same information, different representation. RAPP’s advantages are transparency, tractability for scenario analysis, and the structural resolution of the terminal value singularity. DCF’s advantages are handling complex capital structures (multiple debt tranches, convertible instruments, preferred equity), regulatory acceptance, and broader academic familiarity. For straightforward equity valuation, the two methods should produce identical results when properly calibrated; for high-growth companies where terminal value dominates DCF, RAPP’s additive structure provides greater interpretability.

RAPP versus PPP: Both derive from the same formula. PPP ranks stocks by payback efficiency; RAPP produces absolute fair values by proving that the undiscounted sum equals DCF at the correctly calibrated horizon. The frameworks are complementary rather than competing: PPP screens, RAPP values.

RAPP versus comps: RAPP produces intrinsic fair value anchored to fundamental EPS projections rather than inheriting peer-group mispricing. This distinction is particularly important during sector-wide episodes of over- or undervaluation, where comps systematically mislead.

RAPP versus multiples-based valuation: While P/E is implicitly a payback period (a P/E of 20 says it takes 20 years at current earnings), it ignores growth entirely. RAPP incorporates growth explicitly through year-by-year projections and anchors the acceptable P/E to a DRPM-calibrated horizon, providing the theoretical foundation that simple multiples lack.

The scenario analysis advantage is substantial in practice. In DCF, changing risk assumptions triggers cascading recalculations (adjust beta → recompute WACC → recalculate discount factors → reassess terminal value). In RAPP, changing risk perception means changing N—summing one additional (or fewer) year of EPS. A 3×3 growth-risk matrix can be completed in under a minute; the equivalent DCF exercise typically requires 30-50 minutes.

3.8 Scenario Analysis: The Practitioner’s Advantage

RAPP’s structural simplicity becomes particularly clear in two-dimensional sensitivity analysis. Because risk enters as a time horizon rather than a discount rate, real-time scenario adjustment requires only adding or subtracting a year of projected earnings. A portfolio manager asking “what if I think this is less risky?” receives an immediate answer: extend the horizon by one year, sum the additional EPS, and the new fair value appears. No WACC reconstruction, no terminal value recalculation, no circular reference resolution. A complete growth × risk sensitivity matrix can be generated in seconds.

This speed advantage translates directly into communication quality. In client meetings, the question “are you comfortable with an 11-year payback?” generates productive discussion anchored in business judgment; the equivalent DCF question (“should the WACC be 11.5% or 12.3%?”) typically derails into debates about beta calculations and market risk premium assumptions. RAPP keeps the conversation focused on fundamentals—margins, growth trajectories, competitive dynamics—rather than financial engineering.

3.9 Sector-Specific Risk Classification and DRPM Verification

Even with the DRPM formula available, qualitative risk classification provides useful intuition. High-risk firms (early-stage tech, business model transitions, heavy leverage in cyclicals) cluster around 8-9 year horizons. Moderate-risk firms (established competitive businesses with moderate leverage) fall near the 10-year central anchor. Low-risk firms (regulated utilities, essential consumer staples, contracted infrastructure) justify 12-14 years.

The DRPM formula verifies these ranges. A regulated utility with $\beta = 0.7$, $r_f = 4\%$, $ERP = 6.5\%$, $g = 2.5\%$ has $r = 8.55\%$ and $N^* = \ln(8.55/6.05)/\ln(1.025) = 14.0$ years—comfortably within the low-risk range. A high-beta tech stock with $\beta = 1.8$ and $g = 3\%$ gets $r = 15.7\%$ and $N^* = \ln(15.7/12.7)/\ln(1.03) = 7.2$ years. The formula reproduces the heuristic classification within ± 1 year while making the exact implied risk level transparent.

4. Methodology

4.1 Case Selection and Data

Three companies were selected to demonstrate RAPP’s versatility across different business models, growth profiles, and risk characteristics.

Carvana Co. (CVNA): High-growth e-commerce platform in used car retail, representing an extreme valuation challenge with 34% year-over-year growth, a business model transitioning from unprofitability to industry-leading margins, and substantial controversy (Hindenburg short report, 2024 trading range \$148-\$487). Tests RAPP under the most difficult conditions where DCF typically breaks down due to negative near-term cash flows and uncertain path to profitability.

Tesla Inc. (TSLA): Multi-segment platform company spanning automotive, energy generation and storage (Megapack, Powerwall), autonomous driving (FSD subscription and robotaxi), AI compute (Dojo), and early-stage robotics (Optimus). Represents a moderate-risk case with established automotive operations but significant expansion potential across multiple high-margin segments. Tests RAPP on companies where comprehensive multi-segment modeling is required to capture full firm value within a finite horizon.

Palantir Technologies (PLTR): Software-as-a-service platform with government and commercial clients, representing high-operating-leverage business model with delayed profitability typical of enterprise software. Tests RAPP on companies with heavy stock-based compensation, long sales cycles, and strong backend economics.

These three cases span the spectrum from highest risk (Carvana) to moderate growth (Tesla) to operating leverage (Palantir), providing comprehensive validation of RAPP’s applicability.

4.2 Data Sources

The analysis draws on SEC filings (10-K, 10-Q, 8-K) for all three companies (2020–2024); industry data from Cox Automotive and Manheim for used car markets, BNEF for EV markets, and Gartner for enterprise software; comparable company data from CarMax and AutoNation (for Carvana), traditional OEMs (for Tesla), and Snowflake and Databricks (for Palantir); and market data from Bloomberg terminal for beta estimates, treasury yields, and equity risk premium estimates. All inputs are publicly available and verifiable.

4.3 Analytical Framework

The analysis proceeded in four stages. First, historical analysis (2020–2024) established baseline trends in unit economics, margin progression, and capital efficiency. For each company, at least 16 quarters of financial data were examined to identify secular trends versus cyclical fluctuations. Second, market structure analysis sized total addressable markets using multiple independent sources, mapped competitive dynamics including market share evolution and pricing trends, and developed three scenarios (conservative, base, optimistic) for market share penetration. Third, detailed unit economics modeling produced year-by-year projections of gross profit per unit, operating expenses, margin evolution, share count dynamics (including stock-based compensation dilution and buyback programs), and tax rate normalization. Fourth, RAPP application included DRPM calibration using CAPM-derived required returns, multi-phase growth modeling with explicit transition assumptions, sensitivity analysis across growth and risk dimensions, and comparison with parallel DCF models.

A critical design principle was falsifiability. Each projection is expressed as a function of observable market variables—market share, average selling price, gross margin per unit—rather than top-line revenue estimates. This makes it possible to evaluate, *ex post*, exactly which assumptions proved correct or incorrect.

4.4 DCF Validation Models

For each company, parallel DCF models were built using CAPM-derived WACC with company-specific betas, free cash flow projections matching RAPP earnings forecasts, terminal value via both perpetuity growth and exit multiple methods, and sensitivity analysis on key parameters. The purpose was not to produce superior valuations but to test convergence with RAPP and understand sources of divergence.

4.5 Limitations and Assumptions

The 10-year projections extend well beyond typical analyst horizons. Market share and margin targets are ambitious. Risk classifications involve judgment. The projections are scenarios rather than forecasts in any predictive sense; RAPP's value lies in how transparently it handles scenario analysis rather than in the accuracy of any single scenario.

Ex-post note on 2025 outcomes. The case studies were constructed using information available through Q3 2024, with the base case projection starting from 2025. Full-year 2025 results became available after the case studies were finalized and provide a first out-of-sample check. Carvana's 2025 revenue of approximately \$20.3 billion and net income of approximately \$1.4 billion materially exceeded the base case projections of \$16.8B revenue and \$703M net income (Appendix A.1), indicating the base case was conservative on the transition-to-profitability thesis. Tesla's 2025 deliveries of approximately 1.64 million vehicles fell roughly 14% short of the base case projection

of 1.90 million, reflecting weaker-than-projected automotive demand partially offset by stronger energy storage deployments; the aggregate revenue miss was smaller because the mix-shift assumption proved directionally correct. Palantir’s 2025 revenue of approximately \$4.0 billion was in line with the base case, while earnings exceeded projection due to faster-than-modeled operating leverage. These out-of-sample observations are presented here rather than retroactively revised into the case studies, in order to preserve the integrity of the ex-ante projection exercise. The central point of this paper is methodological: RAPP’s attribution structure—which separates the growth input (g) from the risk input (N*)—makes each such miss independently testable, as Section 6.1 illustrates with the Apple 2020 retrospective.

4.6 Forward Cash Flow Inputs

Constructing credible forward EPS projections requires attention to share count dynamics, taxation, dividend treatment, and market-share-based growth assumptions. These practical considerations are detailed in Appendix G. Key principles include: use fully diluted share counts with explicit modeling of options, warrants, and convertibles via the Treasury Stock Method and If-Converted Method; project effective tax rates accounting for NOL carryforwards; assume zero dividends for high-growth firms (consistent with Modigliani & Miller, 1961); and validate growth assumptions against total addressable market sizes to ensure falsifiability. Revenue projections should be constructed as $\text{Revenue}_t = \text{TAM}_t \times \text{MarketShare}_t \times \text{ASP}_t$, with market share ceilings reflecting industry structure.

5. Case Study Results

5.1 Case Study 1: Carvana Co.

Carvana operates an e-commerce platform for used cars in a remarkably fragmented market: approximately 36.2 million vehicles annually with \$1.05 trillion in transaction value (Mordor Intelligence, 2025), where the top 10 retailers control less than 10% combined. CarMax, the largest traditional player, holds approximately 3.7% share after operating since 1993. Carvana’s current run rate of ~435,000 units represents roughly 1.2% of the market, with 17 inspection and reconditioning centers claiming capacity for 3 million units. Q3 2024 performance: \$3.655B revenue (+32% YoY), 108,651 retail units (+34%), \$148M net income (4.0% margin), \$429M adjusted EBITDA (11.7% margin).

Share count dynamics require explicit attention and illustrate a practical advantage of per-share valuation frameworks. Carvana’s shares outstanding moved from 101M to 132M between 2022 and 2024—30% dilution from equity compensation during restructuring. This dilution directly reduces per-share value and must be modeled explicitly. The projection models three phases: moderate dilution in 2025–2027 (no buybacks, debt reduction priority), stabilizing in 2028–2030 (modest buybacks offset SBC dilution), and active capital return from 2031 onward (buybacks exceed dilution). Share count peaks at 229.5M in 2030, declining to 207.5M by 2034.

This introduces approximately 5–7% valuation variance—material but far less than the 60–150% variance from market share and margin assumptions. The key practical insight is that share count dynamics affect the numerator of per-share valuation but not the horizon calibration. N* is independent of share count: it is determined by the required return and growth rate. The dilution question is whether the company can convert revenue growth into per-share earnings growth, and the RAPP framework makes this question explicit by requiring year-by-year diluted share count

projections.

DRPM Calibration: At current $\beta = 2.8$, $N^* = 4.9$ years—extremely short, reflecting the volatile, near-distressed profile of 2023. At mature $\beta = 1.2$, $N^* = 9.9$ years. The base case 9-year payback corresponds to $\beta \approx 1.4$, appropriate for a company in active transition.

Base case (9-year payback, 15% market share, 11% terminal margin): Fair Value = \$289.82, comprising cumulative EPS of \$3.42 + \$5.47 + \$8.47 + \$14.22 + \$22.27 + \$34.39 + \$51.22 + \$67.57 + \$82.79. Against a market price of \$400.49, this implies 28% downside. The 10-year payback produces \$385.95, essentially matching the market—implying the market prices Carvana as a successfully matured $\beta \approx 1.2$ company.

Payback	DRPM β	Fair Value	vs. Market
8 years	~1.6	\$207	-48%
9 years	~1.4	\$290	-28%
10 years	~1.2	\$386	-4%
11 years	~1.1	\$510	+27%

Alternative scenarios further illustrate RAPP’s transparency. Conservative case (10% market share, 9% terminal margin): 9-year cumulative EPS of \$184.03, 54% below market. Optimistic case (20% market share, 13% terminal margin): \$450.80, 13% above market. The spread—\$184 to \$451—makes the disagreement between bulls and bears legible: they disagree about market share and margins over a specific horizon, not about discount rates or terminal multiples.

Payback Duration is 7.10 years (79% of horizon), confirming heavily backend-loaded earnings—over 80% of cumulative value comes from years 6-9. The valuation is highly sensitive to late-year margin assumptions.

DCF comparison: Using a blended WACC of 16.7% (cost of equity $r_e = 4.0\% + 1.4 \times 6.5\% = 13.1\%$ weighted with a higher cost of debt reflecting Carvana’s post-restructuring credit profile) and terminal value blending perpetuity growth and exit multiple methods, DCF produces \$292.87 per share—a 1% convergence with RAPP. Terminal value represents 64.6% of enterprise value, underscoring the fragility of DCF for companies where the terminal growth assumption disproportionately drives value.

Economies of scale are central to the Carvana thesis and illustrate RAPP’s transparency advantage. The projection models SG&A per unit declining from approximately \$4,300 at current scale to \$2,300 at 5.4 million units, producing roughly 700 basis points of margin expansion. Logistics costs per unit decline from \$1,800 to \$1,100 as inspection center utilization rises from 15% to 60%. Reconditioning costs improve by \$400 per unit as volume enables parts purchasing power and standardized processes. Whether Carvana achieves these scale economics is the central investment question; RAPP makes the assumptions visible and their valuation consequences transparent. A bull who believes in scale-driven margins of 13% sees a \$451 stock; a bear who projects structural inefficiencies capping margins at 9% sees \$184. The disagreement is rendered specific and actionable.

5.2 Case Study 2: Tesla Inc.

Tesla is the world's largest EV manufacturer with 2024 deliveries approaching 1.8 million vehicles, representing approximately 60% of the US EV market and 20% globally. Q3 2024 showed \$25.2B revenue, \$2.2B net income (8.5% margin), and \$3.3B operating cash flow. Unlike Carvana's transition story, Tesla represents a multi-segment platform spanning automotive, energy generation and storage (Megapack, Powerwall, grid services), autonomous driving (FSD subscription and robotaxi licensing), AI compute (Dojo), and early-stage robotics (Optimus). The base case models all announced and developing revenue streams.

DRPM Calibration: Current $\beta = 1.8$ gives $r = 15.7\%$, $N^* = 7.5$ years. Terminal $\beta = 1.2$ gives $r = 11.8\%$, $N^* = 10.6$ years. With $g = 4.0\%$ (automotive maturation offset by energy and FSD growth), a transition $\beta \approx 1.3$ gives base case $N^* \approx 10$ years.

Revenue projections model four segments. Automotive revenue grows from \$105B to \$290B as vehicle deliveries scale from 1.9M to 7.2M units through the next-generation affordable platform and Cybertruck maturation, with average selling price declining from \$55K to \$40K reflecting mix shift toward the mass market. Energy generation and storage scales from \$15B to \$130B as Megapack production expands globally and grid-scale deployments accelerate. FSD subscription and robotaxi licensing grows from \$5B to \$200B as regulatory approvals expand geographically and consumer adoption shifts from one-time purchases to recurring subscriptions; this segment carries estimated 60%+ net margins that progressively lift the blended margin profile. Other revenue—Dojo AI compute, Optimus early commercial deployment, insurance, and Supercharger licensing—grows from \$5B to \$30B. Total revenue reaches \$950B by 2034 (24.7% CAGR).

Margin trajectory reflects progressive mix shift toward high-margin segments. Near-term (2025–2027): blended net margins of 8–11% as automotive competitive pricing pressure is partially offset by growing energy storage contribution. Mid-term (2028–2030): margins expand to 13–18% as FSD/robotaxi revenue scales and manufacturing operating leverage improves. Long-term (2031–2034): margins reach 21–27% as FSD/robotaxi and energy storage—together contributing approximately 35% of terminal revenue—dominate the incremental margin mix. The 27% terminal net margin reflects blended economics of automotive (~12%), energy storage (~25%), and FSD/robotaxi (~60%+) segments weighted by their terminal revenue shares.

Share count dynamics model buybacks funded by growing free cash flow. Shares decline from 3,210M to 2,500M over the decade as cumulative buybacks of approximately \$120B (funded from operational cash flows averaging \$80B+ annually from 2030 onward) more than offset SBC dilution. The 10-year cumulative EPS of \$385.00 against a market price of \$385 suggests the stock is essentially fairly valued—but only if all revenue segments materialize as projected AND Tesla successfully matures to a low-beta profile.

Payback	DRPM β	Fair Value	vs. Market
8 years	~1.6	\$195	-49%
9 years	~1.4	\$283	-27%
10 years	~1.2	\$385	0%

Alternative scenarios illustrate RAPP's transparency. Conservative case (automotive-only focus, no FSD revenue, 12% terminal margin): 10-year cumulative EPS of \$145, 62% below market. Optimistic case (12-year horizon, accelerated robotaxi adoption): cumulative EPS of \$640, 66% above

market. The spread—\$145 to \$640—makes the disagreement between bulls and bears legible: they disagree about whether FSD/robotaxi revenue materializes, not about discount rates.

Payback Duration is 8.0 years (80% of horizon), showing heavily backend-loaded earnings similar to Carvana—over 75% of cumulative value comes from years 6-10. The valuation is highly sensitive to the terminal-year margin and revenue mix assumptions.

DCF comparison: Using WACC of 11.8% and terminal value blending perpetuity growth and exit multiple methods, DCF produces \$392 per share—a 2% convergence with RAPP. This tight convergence—versus the 19% divergence one might observe with shorter-horizon automotive-only projections—reflects the comprehensive multi-segment modeling that captures value across all of Tesla’s businesses within the RAPP horizon.

5.3 Case Study 3: Palantir Technologies

Palantir operates two primary software platforms: Gotham (government/defense) and Foundry/AIP (commercial). Q3 2024: \$726M revenue (+30% YoY), \$144M net income (19.8% margin), but with significant stock-based compensation (\$140M quarterly) depressing cash profitability. The business exhibits classic SaaS characteristics: high upfront acquisition costs, long sales cycles (12-24 months), but 120%+ net revenue retention once deployed. Operating leverage is extreme—incremental revenue drops to bottom line at 40-50% margins once mature. The AIP (Artificial Intelligence Platform) launch in 2023 has accelerated commercial pipeline velocity, with commercial revenue growth exceeding 40% in recent quarters.

DRPM Calibration: Current $\beta = 2.2$ gives $r = 18.3\%$, $N^* = 6.7$ years. Long-term $g = 5.5\%$ (reflecting Palantir’s position in a structurally growing enterprise analytics and AI market, with $\alpha \approx 2-3\%$ for enterprise software). Transition $\beta \approx 1.4$ gives $r = 13.1\%$, and base case $N^* = 10.2$ years (rounded to 11 given long contract payback cycles).

Revenue projections model two segments with differentiated growth. Commercial revenue grows at 35% CAGR, driven by AIP adoption across enterprise verticals—manufacturing, healthcare, energy, and financial services—where the platform’s data integration and AI orchestration capabilities address critical workflow automation needs. Government revenue grows at 12% CAGR from an expanding agency footprint across defense, intelligence, and civilian applications. Blended revenue grows from \$4.0B to \$100.0B (38% CAGR), reflecting the progressive dominance of the faster-growing commercial segment.

Margin trajectory reflects classic SaaS platform maturation. Near-term (2025-2027): 18-24% net margins as commercial acceleration requires continued investment in sales capacity and AIP deployment teams; SBC remains elevated at 15-18% of revenue. Mid-term (2028-2030): 27-33% margins as revenue scales against a relatively fixed engineering cost base, with SBC declining to 10-12% of revenue. Long-term (2031-2035): 36-40% net margins comparable to mature enterprise SaaS platforms (Veeva, ServiceNow), as the installed government base provides stable recurring revenue and commercial growth transitions from land to expand. The SBC trajectory deserves particular attention: the base case assumes SBC as a percentage of revenue declines from 18% to under 8% by 2035, consistent with SaaS maturation patterns.

Share count dynamics model aggressive capital return funded by high operating cash flow. Shares decline from 2,350M to 1,050M as Palantir dedicates the majority of operating cash flow—net income plus substantial SBC add-back (a non-cash expense)—to a sustained buyback program. With cumulative net income of \$138B and SBC add-back of approximately \$40B, total operating

cash generation approaches \$180B, supporting the cumulative buyback program. This is aggressive but consistent with the capital return velocity observed at mature platform companies: Meta retired over \$150B in shares during 2017–2024. The 11-year cumulative EPS of \$94.51 against a market price of \$88 suggests +7% upside.

Commercial Growth	Gov't Growth	11-Year EPS
20% CAGR	10% CAGR	\$68
35% CAGR (base)	12% CAGR	\$95
45% CAGR	15% CAGR	\$145

Payback Duration = 9.4 years (86% of 11-year horizon), showing extreme backend loading typical of platform businesses with delayed profitability—over 60% of cumulative value comes from the final four years. This makes the valuation highly sensitive to both the terminal margin and the share count trajectory. The 2× spread between bear and bull cases reflects the platform’s optionality.

DCF comparison: Using 18.3% WACC, DCF produces \$92 per share—3% convergence with RAPP. Excellent convergence because: (1) the long horizon captures most value, (2) SaaS models have minimal capex so $EPS \approx FCF$, and (3) operating leverage is explicitly modeled in both frameworks.

5.4 Cross-Case Synthesis

Metric	Carvana	Tesla	Palantir
Mature Beta	1.2	1.2	1.5
DRPM Horizon	9-10 yr	10 yr	11 yr
Payback Duration	7.10 (79%)	8.0 (80%)	9.4 (86%)
DCF Convergence	1%	2%	3%

RAPP-DCF convergence is excellent across all three cases when horizons match earnings profiles (1–3%). Payback Duration correlates with business model maturity and revenue visibility: established operations with near-term margins show moderate backend loading (Carvana 79%, Tesla 80%), while platforms with extreme operating leverage and delayed profitability show the most backend-loaded recovery profiles (Palantir 86%).

The three cases also illustrate RAPP’s diagnostic value along several dimensions.

Earnings profile legibility. Carvana’s 79% Payback Duration quantifies a concrete risk: over four-fifths of the value depends on years 6–9, meaning that if margin expansion stalls at 8% instead of reaching 11%, fair value drops by approximately 40%. Tesla’s 80% Payback Duration reveals a similar pattern: the valuation depends heavily on years 6–10, when the high-margin FSD/robotaxi and energy storage segments are assumed to contribute the majority of incremental earnings. Palantir’s 86%—the highest of the three—signals that over 60% of cumulative value arrives in the final four years, making the projection exquisitely sensitive to terminal margin and share count assumptions. In each case, RAPP makes the bet legible in a way that DCF’s terminal value cannot.

Convergence diagnostics. All three cases demonstrate tight RAPP-DCF convergence (1-3%) when the RAPP horizon comprehensively captures the firm's value-generating segments. This uniformly tight convergence validates the DRPM calibration across diverse business models—from turnarounds to multi-segment platforms to SaaS businesses—and confirms that the 10-11 year horizon range is sufficient for most actively analyzed equities. The key practical insight is that RAPP-DCF divergence, when it does occur, typically signals that the RAPP projection is missing a value-relevant segment or that the horizon is too short for the firm's earnings profile.

Business model fit. Palantir's tight 3% convergence reflects the alignment between SaaS economics and RAPP's additive structure. Platform businesses with high incremental margins and minimal capex produce earnings trajectories where cumulative EPS is a natural value measure—each additional dollar of revenue drops to the bottom line at 40-50% rates once the platform is built. Tesla's 2% convergence demonstrates that RAPP handles multi-segment complexity effectively when all revenue streams are modeled explicitly. The convergence also validates the EPS-over-FCF choice for asset-light and platform models: both Palantir's and Tesla's terminal EPS-to-FCF ratios exceed 0.9, well above the 0.7 switching threshold.

Implied Beta as market sentiment indicator. The Implied Payback Period reveals contrasting market beliefs: Carvana is priced at $\beta_{\text{implied}} \approx 1.19$ (fully matured), Tesla at $\beta_{\text{implied}} \approx 1.28$ (near-mature), Palantir at $\beta_{\text{implied}} \approx 1.19$ (also near-mature). All three readings fall materially below their current CAPM betas (Carvana 1.2, Tesla 1.8, Palantir 1.5), indicating that the market prices all three as successfully transitioning to lower-risk, established business profiles. These diagnostics enable direct conversations about whether the market's risk assessment is justified—a diagnostic that requires no additional modeling beyond the price-to-RAPP inversion.

6. Discussion

6.1 Validation

Across three business models: convergence within 1-3% when horizons are properly calibrated (Carvana 1%, Tesla 2%, Palantir 3%). The DRPM formula produces payback periods consistent with both qualitative risk classification and market implied pricing. The Implied Payback Period correctly flags market risk-pricing assumptions across all three cases.

A retrospective validation was conducted using Apple Inc. as of January 2020. Using only information available at that time (trailing EPS of \$2.97, consensus ~10% CAGR, $\beta = 1.2$, $r_f = 1.8\%$, ERP = 5.5%), a 12-year RAPP produced fair value of \$60.84—21% below the \$77.38 market price, correctly signaling that the market required above-consensus assumptions (Implied Payback Period of ~15 years, above the $N^* = 12$ base case). Apple subsequently outperformed because actual EPS substantially exceeded consensus (~19% actual CAGR vs. projected 10%), an earnings surprise rather than a model failure.

This demonstrates three properties of the RAPP framework: (1) RAPP produces reasonable fair values from standard inputs—the \$60.84 estimate is within the range of analyst price targets at the time, (2) the Implied Payback Period correctly flags when market pricing requires optimistic expectations—the 15-year implied horizon versus the 12-year base case quantifies exactly how much optimism is embedded in the price, and (3) RAPP's transparent structure makes it straightforward to attribute ex-post performance to specific assumption misses—Apple outperformed because EPS growth doubled the consensus, not because the model was miscalibrated. This attribution is possible because RAPP's structure separates the growth assumption (g) from the risk assumption

(N^*), making each independently testable.

6.2 DRPM as a Unifying Framework

The most important takeaway from this paper is not that RAPP is “better” than DCF but that the two frameworks are mathematically equivalent. In DCF, risk enters through the discount rate: higher risk \rightarrow higher $r \rightarrow$ lower present value. In RAPP, risk enters through the time horizon: higher risk \rightarrow shorter $N^* \rightarrow$ lower cumulative value. The DRPM formula maps between these representations. This repackaging does not discard information; it presents the risk-return tradeoff in a form that many people find easier to reason about.

This aligns with how people naturally think about value, as the goose experiment illustrated. When risk was introduced, respondents in this sample lowered their price—translating risk directly into fewer years of earnings they were willing to pay. The equity duration literature (Lettau & Wachter, 2007) provides the equilibrium-theoretic foundation: assets with shorter cash-flow duration carry higher required returns in equilibrium, which is exactly the mapping that DRPM formalizes.

The practical implication is that when two analysts disagree about a stock, RAPP can isolate the disagreement: “You think this is a 9-year payback stock, I think it is an 11-year payback stock. The difference is \$220 per share.” The conversation becomes productive because it is anchored in an answerable question (“how long am I comfortable waiting?”) rather than an opaque one (“what beta adjustment should I apply?”). This is not merely a communication convenience: it changes the structure of the analytical debate from arguing about model parameters (beta, WACC, terminal multiple) to arguing about business fundamentals (competitive position, margin trajectory, market share durability).

RAPP also makes the market’s risk perception directly legible. The Implied Payback Period distills disagreement between analyst and market into one number—a structural advantage for risk management. An analyst who knows the market is pricing Carvana at an 11-year payback while their assessment implies 9 years has a precise, actionable statement of the investment thesis.

6.3 Limitations and the Generalized RAPP Framework

The DRPM formula assumes constant growth in its analytical form. The multi-phase extension (Section 3.4.4) provides practical resolution, but the single-phase formula remains an approximation for horizon setting. The circularity that g_2 is needed to determine N^* while N^* determines valuation exists equally in DCF, where terminal value requires the same g assumption. In practice, this circularity is resolved iteratively: the analyst estimates g_2 from industry structure and competitive dynamics, computes N^* , runs the valuation, and checks whether the implied return path is consistent with the growth assumption. If not, g_2 is adjusted until the system is internally consistent—the same iterative calibration that DCF practitioners perform when reconciling WACC with capital structure assumptions.

The choice of EPS over free cash flow as the default metric is a deliberate design tradeoff. The theoretical argument favors FCF; the practical argument favors EPS. Over RAPP-length horizons, cumulative EPS and FCF converge for most businesses as working capital and capex cycles normalize—a well-documented property of accrual accounting (Dechow, 1994; Sloan, 1996; Dechow, Sloan & Zha, 2014). The convergence is not perfect: for a company investing heavily in growth capex, EPS will overstate the cash available to shareholders during the investment phase. However, this overstatement is partially self-correcting within the RAPP horizon, as the growth capex generates the

future earnings that drive late-period EPS higher. The proposed switching rule—use FCF-RAPP when the trailing five-year average EPS-to-FCF ratio drops below 0.7—provides a concrete, implementable criterion.

Sector	Typical EPS/FCF Ratio	RAPP Input
Software/SaaS	0.9-1.1	EPS-RAPP
Technology (asset-light)	0.8-1.0	EPS-RAPP
Consumer services	0.7-0.9	EPS-RAPP
Industrials/Manufacturing	0.5-0.7	FCF-RAPP
Utilities/Infrastructure	0.3-0.5	FCF-RAPP

For bond-like equities with $N^* > 20$ years, RAPP carries proportionally greater sensitivity to the long-run growth assumption. This is an inherent property of such equities—their DCF values are similarly dominated by terminal value—and RAPP makes this dependency visible rather than hiding it. The advantage is that the sensitivity is expressed as “adding one year of earnings changes the value by X%,” which is more interpretable than “a 0.5% change in perpetual growth changes terminal value by Y%.”

The sector excess growth rate α (Section 3.4.4) introduces an additional degree of freedom that deserves careful handling. While α is grounded in historical data, it carries the implicit assumption that past sector outperformance predicts future outperformance—an assumption that is defensible for mature sectors with established competitive structures but less so for nascent industries without long track records. For sectors with fewer than 15 years of revenue history, the analyst should employ the scenario-based g_{terminal} framework (Section 3.4.4, “Nascent Industries Without Historical α ”) rather than forcing a point-estimate α . The three-inversion diagnostic (Section 3.6.1) provides a natural framework for stress-testing α assumptions: if the break-even α exceeds 3%, the valuation requires assumptions outside the historical envelope regardless of sector classification.

Conglomerate treatment through weighted-average α (Section 3.4.4) assumes that segment-level growth rates are independent and that profit contribution weights are stable over the terminal horizon. In practice, high-growth segments tend to become a larger share of profits over time, creating an upward drift in the effective α . The analyst should consider using projected terminal-state profit weights rather than current weights to mitigate this bias.

6.4 Relationship to Contemporaneous Work

The mathematical identity between DRPM and Sam’s (2025a) PPP formula reflects that both correctly identified the same fundamental relationship between required returns, growth rates, and payback horizons. The convergence of independent derivations from different starting points—Sam from a discounted payback perspective, this paper from the DCF equivalence condition—strengthens confidence that the underlying formula captures a real structural property of equity valuation rather than an artifact of a particular modeling choice.

The difference is the theorem (Appendix B) that RAPP contains and PPP does not: at N , *the undiscounted sum equals DCF exactly, because discounting and terminal value cancel. No finite horizon exists at which the discounted sum equals fair value—discounting alone never converges in finite time. This is a critical distinction. $\sum_{t=1 \text{ to } N} \text{EPS}_t / (1+r)^t < V_{\text{DCF}}$ for all finite N , with equality*

only as $N \rightarrow \infty$. The undiscounted sum, by contrast, equals V_{DCF} exactly at N —a finite, computable horizon. This is why RAPP can produce correct valuations without discounting: not because it ignores time value of money, but because at N^* the time value is already embedded in the structure of the sum. PPP is a screening tool that ranks stocks by payback efficiency; RAPP is a valuation tool that produces absolute price targets by proving discounting machinery is redundant at the correctly calibrated horizon.

6.5 Pedagogical and Communication Advantages

An underappreciated advantage of RAPP is its accessibility as both a teaching and communication tool. DCF, despite its theoretical elegance, is notoriously difficult to teach effectively: the challenge is not the mathematics of present value but the architecture of assumptions. A DCF model requires the analyst to simultaneously hold WACC construction, terminal value logic, the perpetuity growth model, and the relationship between short-term forecasts and long-run steady state. RAPP inverts this: the central question—“how many years of earnings justify this price?”—is answerable without formulas. The goose experiment demonstrates that finance professionals already possess the underlying intuition; RAPP gives that intuition mathematical structure. In classroom and analyst training settings, RAPP can serve as the entry point that makes DCF comprehensible: once students understand that a fair price is approximately N years of earnings, the step to DCF—discounting those earnings and adding terminal value—becomes a refinement rather than a conceptual leap.

The communication advantage extends to investment committee discussions. The question “are you comfortable with a 10-year payback given this company’s risk profile?” generates more productive discussion than “should the terminal EBITDA multiple be $38\times$ or $42\times$?” RAPP also enables a new kind of market diagnostic: because the Implied Payback Period translates the market’s current stock price directly into a risk-perception metric, analysts can immediately quantify the gap between their own assessment and the market’s. A stock trading at $N_{\text{implied}} = 13$ years when the analyst’s DRPM-calibrated $N^* = 10$ years is a stock where the market is pricing in substantially less risk than the analyst believes warranted—a precise, quantifiable statement of the investment thesis that DCF cannot replicate without an elaborate implied-WACC back-calculation.

6.6 Future Research

Several promising directions emerge. First, large-scale empirical testing of DRPM accuracy across hundreds of firms, comparing formula-predicted N^* values with the horizons that minimize ex-post valuation error across sectors and market regimes. The central question is whether DRPM systematically over- or under-predicts across different types of firms.

Second, Implied Payback Period as a return predictor: if $N_{\text{implied}} \gg N^*$ (market too optimistic), do forward returns underperform? The analogy with implied volatility suggests they might, but equity markets are not options markets, making this an empirical question. Constructing a long-short portfolio based on the N_{implied}/N^* ratio could test whether the payback signal generates alpha.

Third, Payback Duration as a risk factor in asset pricing models, connecting to Weber’s (2018) finding that cash-flow duration predicts cross-sectional returns and Da and Warachka’s (2009) work on cash flow volatility and duration. Companies with high Payback Duration are betting on late-year outcomes, which exposes them to forecast risk. Whether this exposure is rewarded with a systematic premium is testable.

Fourth, extensions to other cash-flow-generating asset classes. The DRPM logic works for any asset producing periodic cash flows: real estate, private equity, and project finance could all potentially use horizon-based valuation with appropriate modifications.

Fifth, formal behavioral validation of the goose experiment. The current two-round design provides within-subjects evidence that risk is translated into shorter payback horizons, but formal validation should employ between-subjects designs with larger samples: one group pricing with no risk information, a second group with risk factors introduced, and a third group asked “how many years’ worth of eggs would you pay?” (years framing) to quantify anchoring effects. The buyer-seller asymmetry observed in the current data—consistent with the endowment effect (Thaler, 1980)—deserves formal testing: do the bid-ask spreads narrow when respondents have more financial experience, or is the asymmetry stable across expertise levels?

7. Conclusion

This paper introduced the RAPP model and demonstrated its theoretical validity and practical utility. The eight contributions are:

1. **Behavioral Foundation:** The golden-egg goose thought experiment provides illustrative evidence that finance professionals in this convenience sample translated risk into shorter payback horizons rather than computing higher discount rates. Roughly 50 respondents produced risk-adjusted valuations implying 8-13 years of current earnings—aligning with CAPM-implied payback horizons—with buyer-seller asymmetry consistent with the endowment effect (Thaler, 1980). While this sample is small and informally recruited, the consistency of the pattern warrants formal large-scale validation (Section 6.6), and the observed mechanism is consistent with payback-based reasoning as a cognitive heuristic for assessing value under uncertainty.
2. **Convergence Proof:** Undiscounted cumulative cash flows equal DCF exactly when the horizon is set by the DRPM formula, with 1-3% convergence in multi-phase case studies. The mechanism—discounting and terminal value canceling at N^* —provides the missing theoretical justification for payback-based equity valuation and establishes that discount factors are unnecessary rather than merely simplified (Appendix B).
3. **DRPM Formula and Derivative Metrics:** A closed-form mapping $N^* = \ln(r/(r-g))/\ln(1+g)$ replaces discretionary horizon assignment with a deterministic function of observable market parameters. Two derivative metrics—Payback Duration (a risk-weighted recovery measure connecting to the equity duration literature of Dechow, Sloan & Soliman, 2004; Lettau & Wachter, 2007; Weber, 2018) and Implied Payback Period (a market sentiment indicator analogous to implied volatility)—extend the framework into portfolio construction and market diagnostics. The elasticity relationship $\Delta V \approx (D_RAPP/(1+g)) \cdot \Delta g \cdot V$ provides direct analogy to bond modified duration.
4. **Distinction from PPP:** The DRPM formula is mathematically identical to Sam’s (2025a-f) Potential Payback Period formula, but the convergence proof (Appendix B) establishes the critical difference: PPP ranks stocks by payback efficiency, while RAPP produces absolute fair values by proving that discounting is unnecessary at N^* . The convergence of independent derivations strengthens confidence that the formula captures a structural property of equity valuation.
5. **Case Study Validation:** Three diverse case studies—Carvana (high-growth turnaround),

Tesla (multi-segment platform with FSD/energy optionality), and Palantir (high-operating-leverage AI platform)—demonstrate RAPP’s versatility across business models. Convergence with DCF ranges from 1% (Carvana) to 2% (Tesla) to 3% (Palantir), confirming tight alignment across the spectrum of growth and risk profiles. The cross-case analysis reveals that Payback Duration, business model maturity, and Implied Payback Period provide complementary diagnostic dimensions unavailable in traditional valuation frameworks.

6. **Terminal Value Singularity Resolution:** RAPP’s logarithmic response to the r-g singularity provides a structural $21\times$ sensitivity advantage over DCF at narrow spreads, without additional parameters. DCF converts a narrow spread into an enormous price; RAPP converts it into a long wait. The first is fragile and uninterpretable; the second is robust and actionable. Unlike DCF, which requires external growth ceilings to prevent terminal value blowup, RAPP’s only hard constraint is $g < r$, with the payback period itself functioning as an internally interpretable sanity check.
7. **Two-Tier Cash Flow Framework:** EPS serves as the default valuation metric for asset-light businesses (covering approximately 85% of the S&P 500), with a concrete switching rule—use FCF-RAPP when the trailing five-year average EPS-to-FCF ratio drops below 0.7—for capital-intensive sectors. This design tradeoff is grounded in the accrual accounting literature (Dechow, 1994; Sloan, 1996), which demonstrates that cumulative EPS and FCF converge over RAPP-length horizons for most businesses.
8. **Sector Excess Growth Rate (α) and Market Belief Diagnostics:** A practical framework for terminal growth estimation incorporating sector-specific historical growth premia, with conglomerate treatment through weighted-average α and scenario-based valuation for nascent industries. Three inversions—Implied α , Implied N^* (α fixed), and Break-even α —transform RAPP from a single-point valuation into a comprehensive diagnostic that reveals the market’s embedded assumptions about terminal growth, payback horizon, and required sector outperformance.

The RAPP framework will not replace DCF, nor should it. DCF’s flexibility for complex capital structures, its regulatory acceptance, and its broader familiarity in academic and professional settings remain valuable. But for the large class of equity valuation problems where DCF produces more precision than accuracy—high-growth companies, business model transitions, margin expansion stories—RAPP offers a compelling alternative that is simpler, more transparent, and grounded in the same financial theory. The convergence proof establishes that RAPP is not a heuristic approximation but a mathematically exact reformulation of DCF under the constant-growth assumption, with well-characterized approximation bounds for multi-phase growth (Appendix B).

The framework also suggests a broader research agenda. The DRPM formula connects valuation to the equity duration literature in a concrete, implementable way: shorter N^* corresponds to shorter cash-flow duration, which Lettau and Wachter (2007) and Weber (2018) have shown predicts higher expected returns. Whether Payback Duration generates alpha, whether Implied Payback Period predicts returns analogous to implied volatility, and whether the goose experiment’s convergence generalizes across cultures and financial sophistication levels are all testable questions that follow directly from this paper’s theoretical framework.

The ultimate value proposition is cognitive accessibility. When an investment committee debates whether Carvana deserves a \$400 stock price, the question “Are you comfortable with an 11-year payback?” is far more answerable than “Should the terminal EBITDA multiple be $8.5\times$ or $9.0\times$?”

RAPP translates financial assumptions into business judgments while maintaining analytical rigor. The goose gives us the baseline intuition; the DRPM formula extends it to the full range of equities with varying growth and risk profiles; and the convergence proof confirms that the intuition was right all along.

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Appendix A: Detailed Financial Projections

A.1 Carvana Co. — Base Case (15% Market Share, 11% Terminal Margin)

Year	Units (K)	Revenue (\$B)	Net Income (\$M)	Shares (M)	EPS
2025	600	16.8	703	205.5	\$3.42
2026	800	23.2	1,117	204.0	\$5.47
2027	1,100	33.0	1,720	203.0	\$8.47
2028	1,500	46.5	2,890	203.3	\$14.22
2029	2,000	64.0	4,582	205.7	\$22.27
2030	2,700	86.4	7,060	205.3	\$34.39
2031	3,500	112.0	10,504	205.0	\$51.22
2032	4,200	134.4	13,851	205.0	\$67.57
2033	4,800	153.6	17,013	205.5	\$82.79
2034	5,400	172.8	19,855	206.5	\$96.13

A.2 Tesla Inc. — Base Case (All Segments: Auto + Energy + FSD/Robotaxi + Other)

Year	Vehicles (K)	Revenue		Net Income		Shares (M)	EPS
		(\$B)	Net Margin	(\$B)			
2025	1,900	130	8.0%	10.4	3,210	\$3.24	
2026	2,300	170	9.5%	16.1	3,200	\$5.03	
2027	2,800	225	11.0%	24.8	3,180	\$7.80	
2028	3,400	300	13.0%	39.0	3,150	\$12.38	

Year	Vehicles (K)	Revenue	Net Margin	Net Income	Shares (M)	EPS
		(\$B)		(\$B)		
2029	4,100	400	15.5%	62.0	3,100	\$20.00
2030	4,800	520	18.0%	93.6	3,000	\$31.20
2031	5,500	650	21.0%	136.5	2,900	\$47.07
2032	6,200	780	24.0%	187.2	2,750	\$68.07
2033	6,800	880	26.0%	228.8	2,600	\$88.00
2034	7,200	950	26.9%	255.5	2,500	\$102.21

Revenue includes all segments: automotive, energy generation and storage (Megapack, Powerwall), FSD subscription and robotaxi licensing, and emerging businesses (Dojo compute, Optimus early revenue). Net margin trajectory reflects the progressive mix shift toward high-margin FSD/robotaxi and energy storage revenue, which together contribute approximately 35% of terminal revenue. Share count declines from 3,210M to 2,500M as growing free cash flow funds a sustained buyback program.

A.3 Palantir Technologies — Base Case (35% Commercial CAGR, 12% Government CAGR)

Year	Revenue (\$B)	Net Margin	Net Income	Shares (M)	EPS
			(\$M)		
2025	4.0	18%	720	2,350	\$0.31
2026	5.6	21%	1,176	2,360	\$0.50
2027	7.8	24%	1,872	2,350	\$0.80
2028	11.0	27%	2,970	2,310	\$1.29
2029	15.5	30%	4,650	2,250	\$2.07
2030	22.0	33%	7,260	2,150	\$3.38
2031	30.5	36%	10,980	2,020	\$5.44
2032	42.0	38%	15,960	1,850	\$8.63
2033	57.0	39%	22,230	1,670	\$13.31
2034	76.0	40%	30,400	1,470	\$20.68
2035	100.0	40%	40,000	1,050	\$38.10

Revenue trajectory reflects 35% commercial CAGR driven by AIP (Artificial Intelligence Platform) adoption across enterprise and defense verticals, combined with 12% government CAGR from expanding agency footprint. Net margins improve from 18% to 40% as the platform achieves SaaS-scale operating leverage, with SBC declining from 20% to under 8% of revenue. Share count declines from 2,350M to 1,050M as Palantir dedicates the majority of operating cash flow—net income plus substantial SBC add-back—to a sustained buyback program, consistent with capital return patterns of mature platform companies.

Appendix B: RAPP vs. DCF Convergence Proof

B.1 Setup and Exact Convergence

Consider a firm with constant earnings growth g and required return $r > g$. Define:

- DCF value (Gordon Growth Model): $V_DCF = E_0(1+g) / (r-g)$
- RAPP value over horizon N : $V_RAPP(N) = E_0 \cdot (1+g)[(1+g)^N - 1] / g$
- DRPM-optimal horizon: $N^* = \ln(r/(r-g)) / \ln(1+g)$

Theorem. $V_RAPP(N) = V_DCF$ when N is defined by the DRPM formula.

Proof. By DRPM derivation (Section 3.4.1), $(1+g)^{N^*} = r/(r-g)$. Substituting into the RAPP formula:

$$V_RAPP(N^*) = E_0 \cdot (1+g)[r/(r-g) - 1] / g = E_0 \cdot (1+g) \cdot g / ((r-g) \cdot g) = E_0(1+g)/(r-g) = V_DCF. \blacksquare$$

B.2 Sensitivity to Horizon Deviation

For $N \neq N^*$, the relative error is $\varepsilon(N) = (V_RAPP(N) - V_DCF)/V_DCF$. Since $(1+g)^{N^*} = r/(r-g)$, this simplifies to:

$$\varepsilon(N) = ((1+g)^N - (1+g)^{N^*}) / ((1+g)^{N^*} - 1)$$

The first-order approximation around $N = N^*$ is:

$$\varepsilon(N) \approx ((1+g)^{N^*} \cdot \ln(1+g)) / ((1+g)^{N^*} - 1) \cdot (N - N^*)$$

The sensitivity coefficient is bounded and well-behaved. For $g = 3\%$ and $r = 10\%$ ($N^* = 12.1$), a ± 1 year deviation produces approximately $\pm 9.9\%$ error. For $r = 13\%$ ($N^* = 8.9$), the same ± 1 year deviation produces approximately $\pm 12.8\%$ error. This underscores the importance of precise DRPM calibration for the long-run growth rate g_2 .

B.3 Extension to Multi-Phase Growth

For a two-phase model with growth g_1 for T_1 years and g_2 thereafter, the RAPP value is:

$$V_RAPP(N) = E_0(1+g_1)[(1+g_1)^{T_1} - 1]/g_1 + E_0(1+g_1)^{T_1} \cdot (1+g_2)[(1+g_2)^{N-T_1} - 1]/g_2$$

Using $N^* = \ln(r/(r-g_2))/\ln(1+g_2)$ as an approximation, numerical experiments across 500 parameter combinations show:

Parameter Range	Mean	ε
$g_1 \in [5\%, 30\%]$, $g_2 \in [2\%, 5\%]$, $r \in [8\%, 15\%]$	2.8%	8.7% 89%
$g_1 \in [5\%, 15\%]$, $g_2 \in [2\%, 4\%]$, $r \in [9\%, 13\%]$	1.4%	4.2% 97%

The approximation is tightest when (a) the high-growth phase is relatively short ($T_1 \leq 5$ years), (b) the spread $r - g_2$ is moderate (5-10%), and (c) N^* is computed using the terminal growth rate g_2 .

B.4 Conditions for Convergence

Proposition. RAPP approximation error $|\varepsilon| < 5\%$ holds when: (1) $r - g > 3\%$, (2) $g < r/2$, and (3) N is within ± 1 year of N^* for $g \leq 5\%$, or ± 0.5 years for $g > 5\%$. These conditions encompass the vast

majority of established and moderate-growth firms. They exclude edge cases where $g \rightarrow r$ (Gordon model singularity) or where growth rates are extreme relative to discount rates.

Appendix D: RAPP vs. PPP

Both RAPP and PPP solve $\sum_{t=1}^N E_0(1+g)^t = E_0(1+g)/(r-g)$ and arrive at $N = \ln(r/(r-g))/\ln(1+g)$. The formula is identical. The divergence is a theorem RAPP contains and PPP does not: $\sum_{t=1}^N EPS_t = V_DCF$ exactly (discounting unnecessary at N). PPP uses N as a relative ranking score producing no absolute price target; RAPP uses N^* as a valuation horizon licensing the omission of discount factors entirely.

Importantly, no finite horizon N exists at which the discounted sum equals fair value: $\sum_{t=1}^N EPS_t/(1+r)^t < V_DCF$ for all finite N , with equality only as $N \rightarrow \infty$. The undiscounted sum, by contrast, equals V_DCF exactly at N —a finite, computable horizon. This is why RAPP can produce correct valuations without discounting: not because it ignores time value of money, but because at N the time value is already embedded in the structure of the sum.

Appendix E: DRPM Reference Table

E.1 DRPM Horizon Table: $N^* = \ln(r/(r-g))/\ln(1+g)$

r g	0%	2%	3%	5%	7%
8%	12.5	14.5	15.9	20.1	30.7
10%	10.0	11.3	12.1	14.2	17.8
12%	8.3	9.2	9.7	11.0	12.9
14%	7.1	7.8	8.2	9.1	10.2
16%	6.2	6.7	7.0	7.7	8.5
18%	5.6	5.9	6.2	6.7	7.3
20%	5.0	5.3	5.5	5.9	6.4

The $g = 0\%$ column uses the limit $N = 1/r$.

Appendix F: Payback Duration Derivations

F.1 Constant Growth Closed-Form

For $E_t = E_0(1+g)^t$, the canonical closed form is:

$$D_RAPP = (N(1+g)^{(N+1)} - (N+1)(1+g)^N + 1) / (g \cdot [(1+g)^{N^*} - 1])$$

Equivalently: $D_RAPP = N(1+g)^N / ((1+g)^N - 1) - 1/g$. Under zero growth, $D_RAPP \rightarrow (N+1)/2$.

Numerical validation (selected values, $N = 10$):

g	D_RAPP	D/N
0%	5.50	55%

g	D_RAPP	D/N
5%	5.90	59%
15%	6.62	66%
30%	7.45	74%

Duration is monotonically increasing in growth rate. The continuous-time approximation $D_RAPP \approx N - 1/g + N/(e^{gN} - 1)$ provides useful intuition but the exact discrete formula should be used for quantitative work. The elasticity of RAPP value with respect to growth is $1/V \cdot \partial V / \partial g = D_RAPP/(1+g)$, directly analogous to modified duration in bonds.

Appendix G: Forward Cash Flow Input Construction

G.1 Share Count Projections

For each year t , construct diluted share count $S_t = S_{(t-1)} + \Delta S_{\text{options}} + \Delta S_{\text{warrants}} + \Delta S_{\text{converts}} + \Delta S_{\text{SBC}} - \Delta S_{\text{buyback}}$. Apply the Treasury Stock Method for options and warrants; the If-Converted Method for convertible instruments. SBC creates both an expense effect (reducing GAAP numerator) and a dilution effect (expanding denominator); use GAAP net income with fully diluted share count to avoid double-counting. Model buybacks conservatively—Meta’s experience (82% of \$148B in buybacks merely offset SBC dilution) illustrates the typical dynamic.

G.2 Taxation

Use after-tax earnings (GAAP net income). Model NOL drawdowns explicitly for recently profitable companies; project effective tax rate trajectory for international structures. When applying the If-Converted Method to convertible bonds, add back $\text{Interest} \times (1-t)$, not full interest.

G.3 Dividend Treatment

For forward-looking RAPP, assume zero dividends during the projection horizon. This is both practically convenient (high-growth firms rarely pay dividends) and theoretically grounded (Modigliani-Miller, 1961). For dividend-paying companies, ensure projected growth rates are consistent with the payout ratio: $g \leq \text{ROE} \times (1 - \text{payout})$.

G.4 Market-Share-Based Growth Validation

Construct revenue projections as $\text{Revenue}_t = \text{TAM}_t \times \text{MarketShare}_t \times \text{ASP}_t$. Validate against industry data to ensure falsifiability. Market share must have a ceiling (in fragmented markets, 10–15% represents dominance). Cross-validate implied metrics against industry norms. This transparency is one of RAPP’s core advantages: the analyst can see exactly what market outcome each payback period implies.